Un peu de géométrie plane élémentaire !

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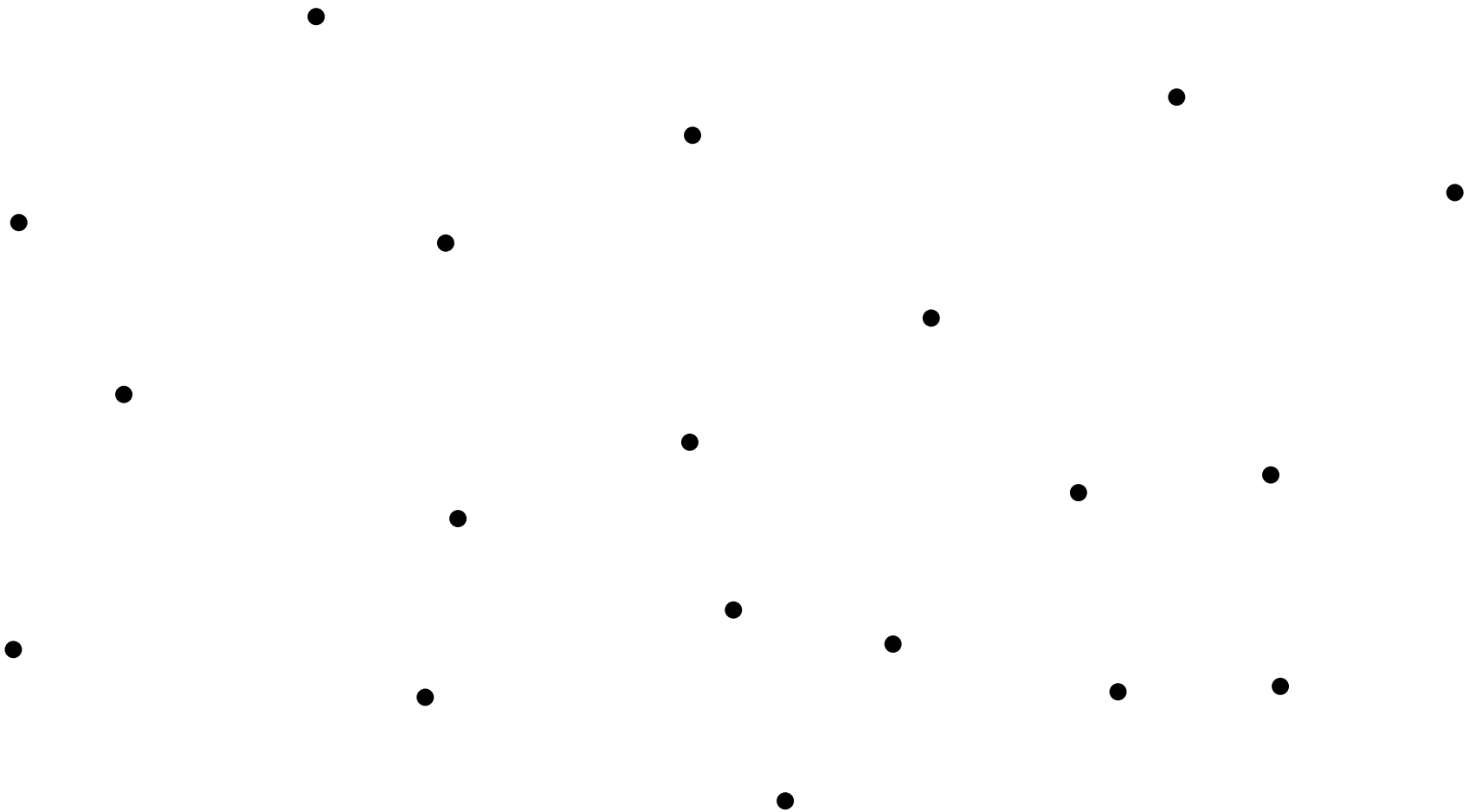
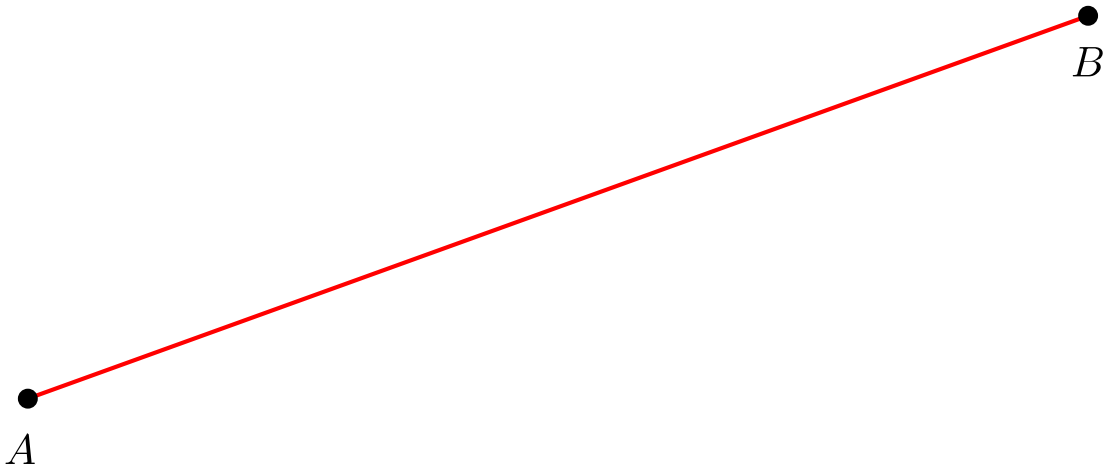
Séances de formation
au
Musée de la Dentelle - Caudry
Les 6 et 13 août 2019

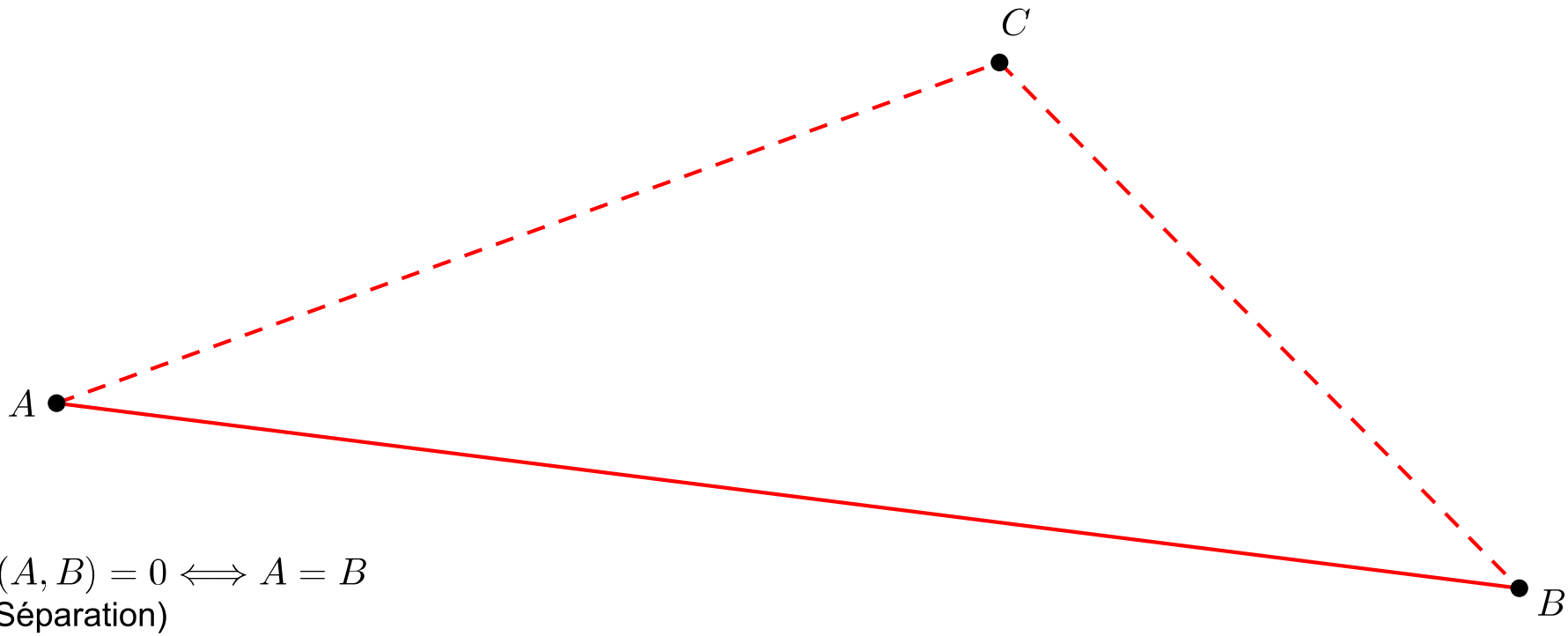
Le plan et ses éléments




A


B


B
A

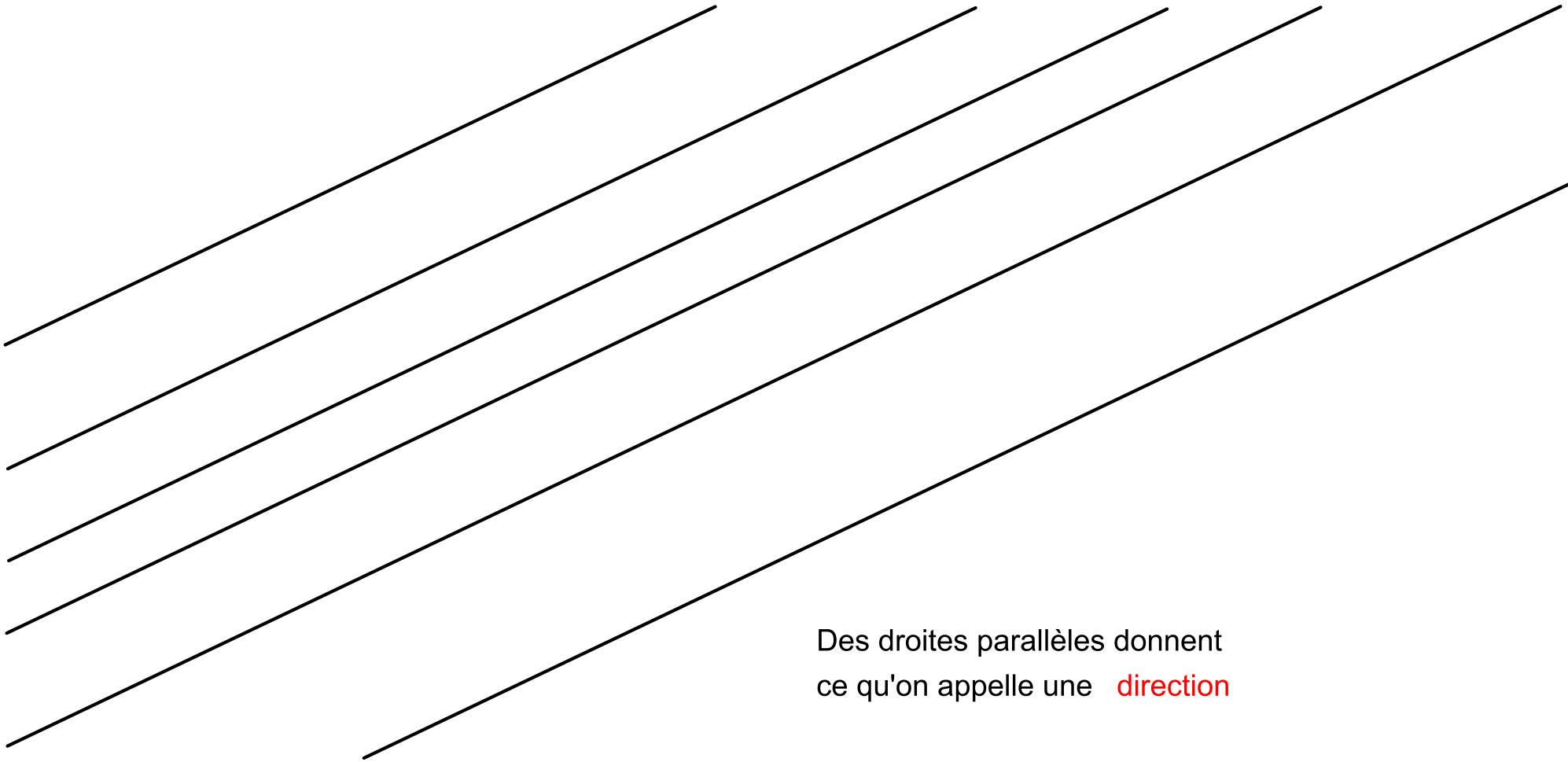


$d(A, B) = 0 \iff A = B$
(Séparation)

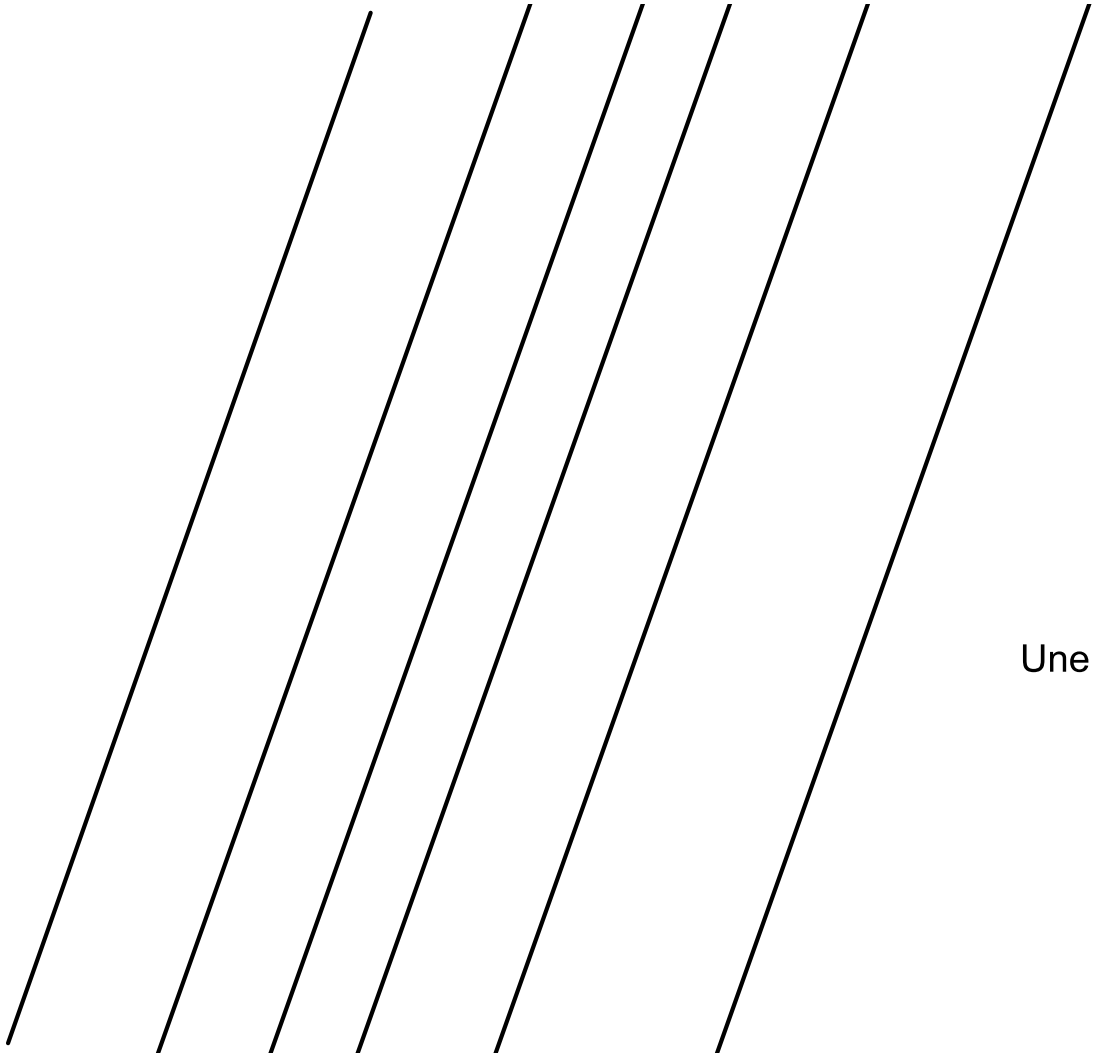
$dA, B) = d(B, A)$
(Symétrie)

$d(A, B) \leq d(A, C) + d(C, B)$
(Inégalité du triangle)

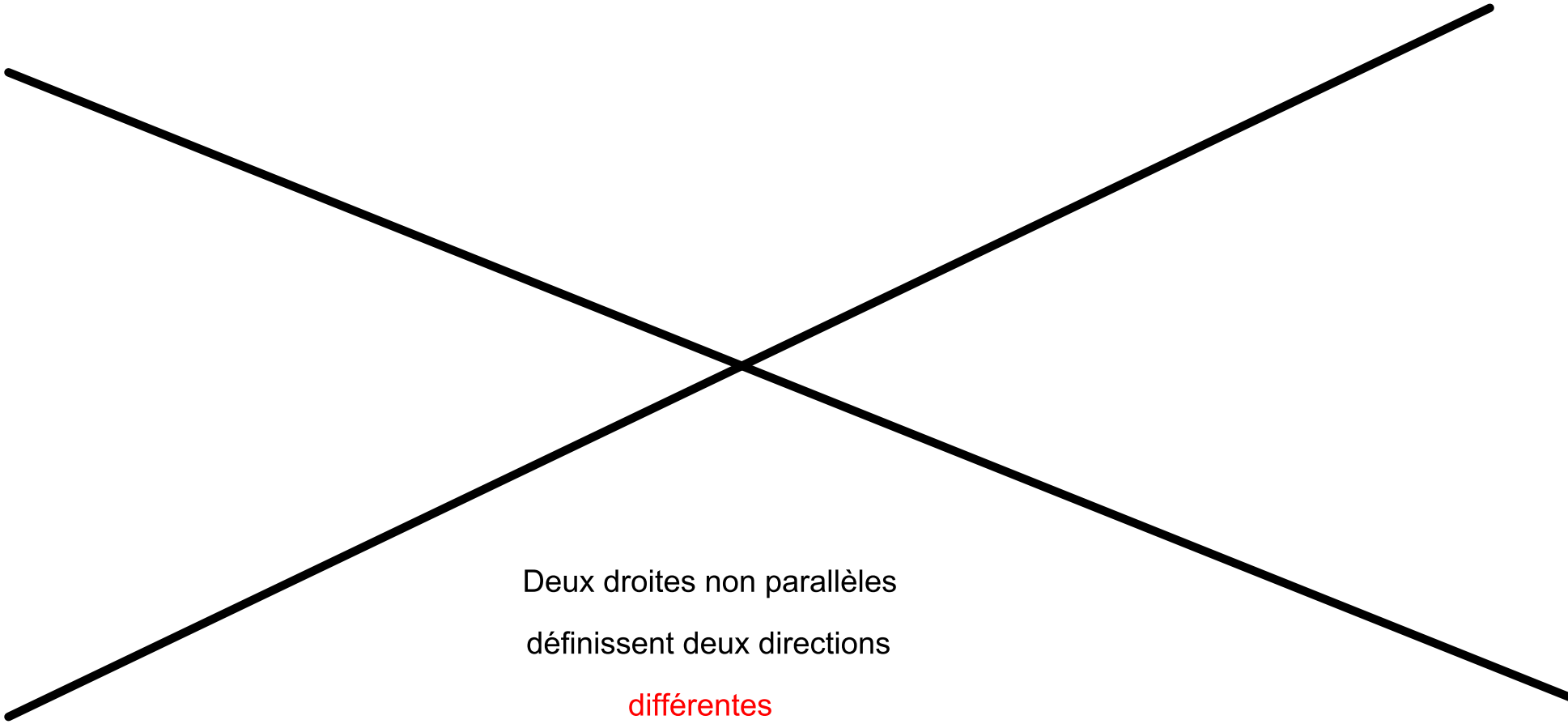
La notion de direction



Des droites parallèles donnent
ce qu'on appelle une direction



Une autre direction

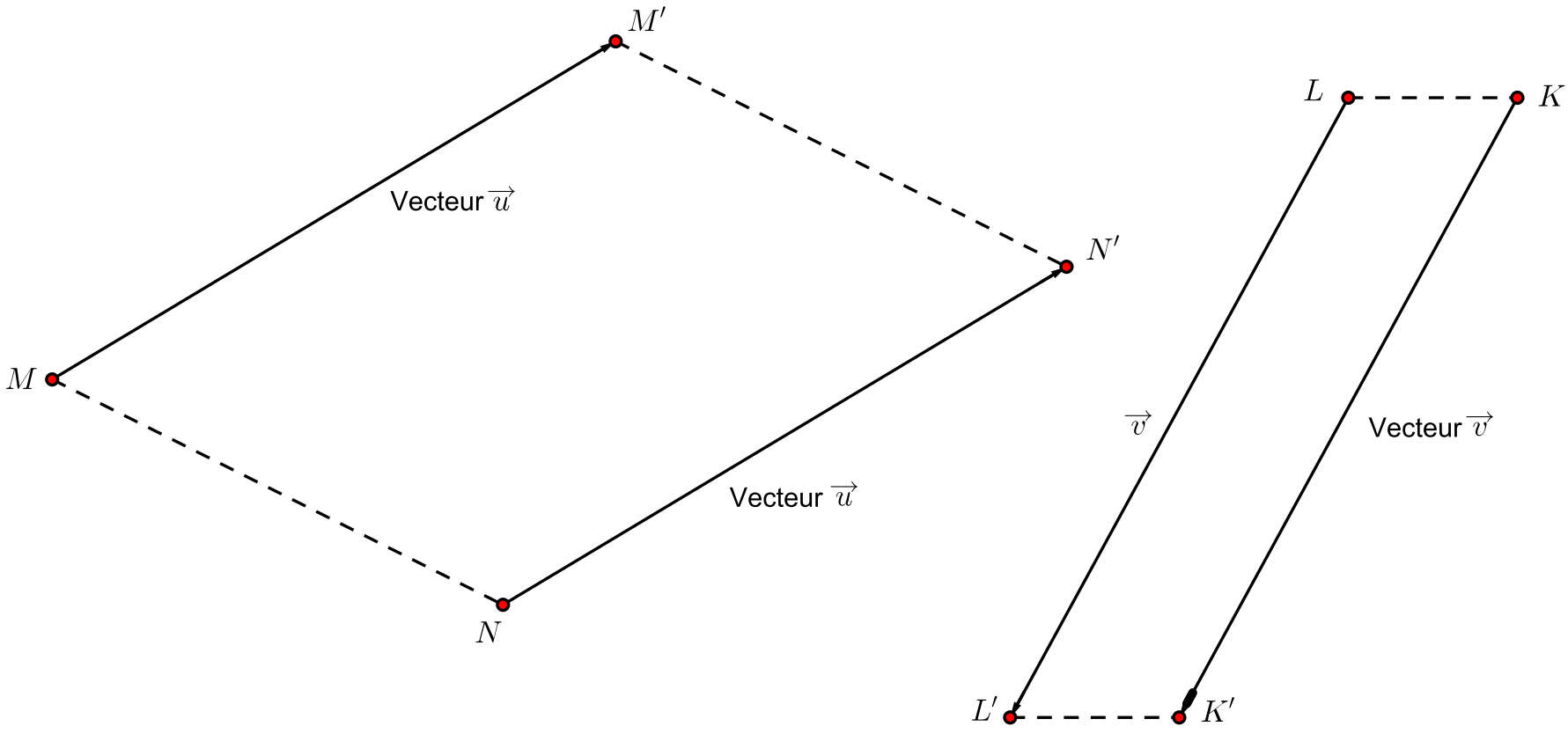


Deux droites non parallèles

définissent deux directions

différentes

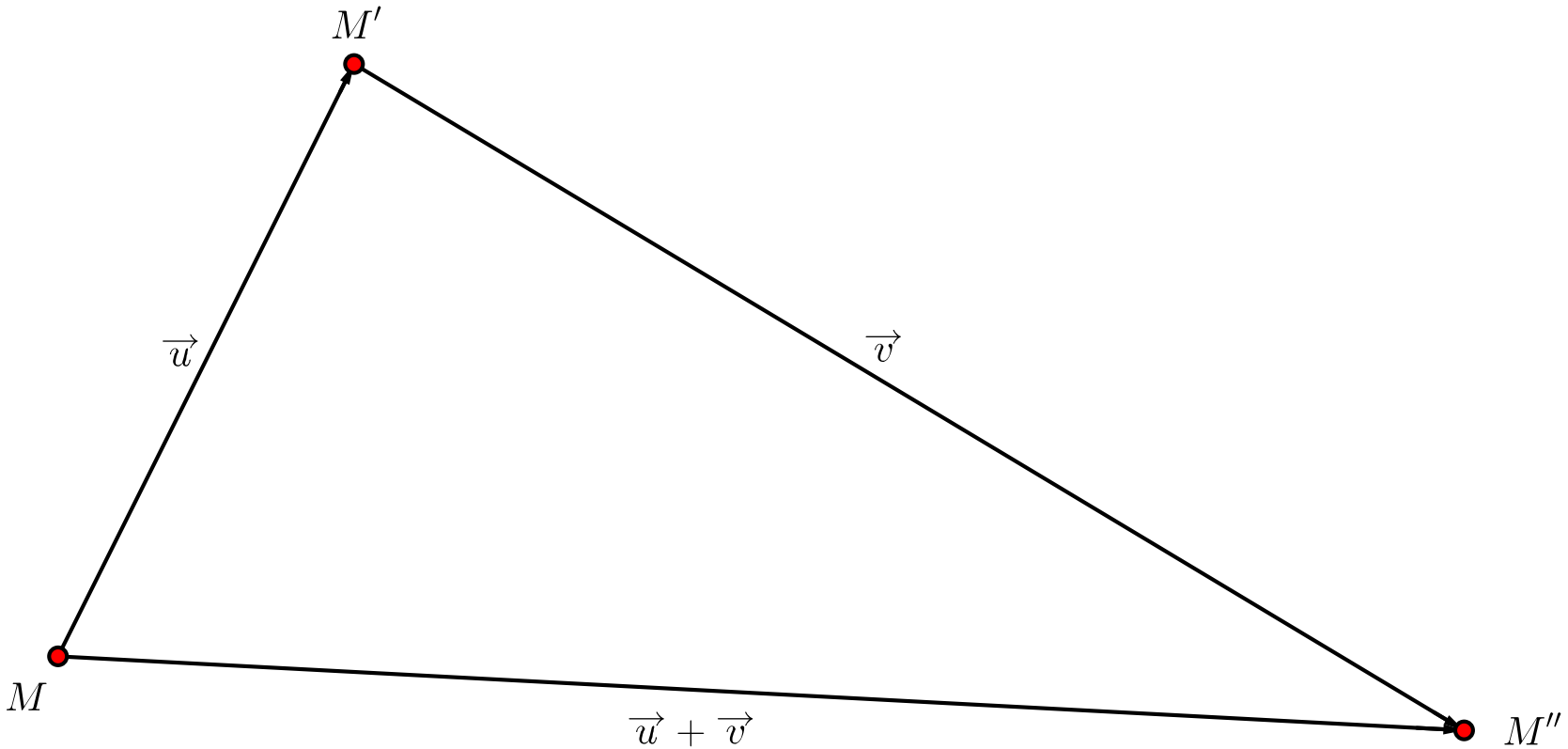
La notion de vecteur



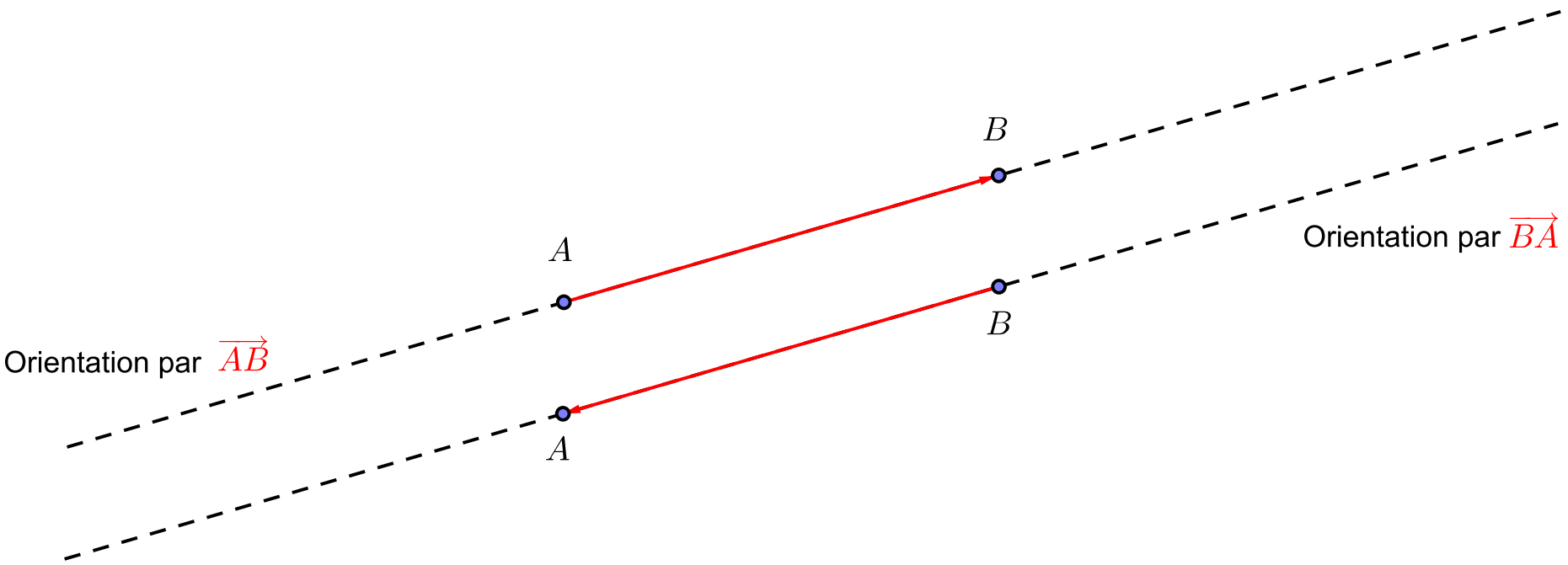
Un vecteur est donc défini par :

- sa *direction* : donnée par une droite qui lui est parallèle ;
- son *sens* ;
- sa *longueur*.

Somme de vecteurs

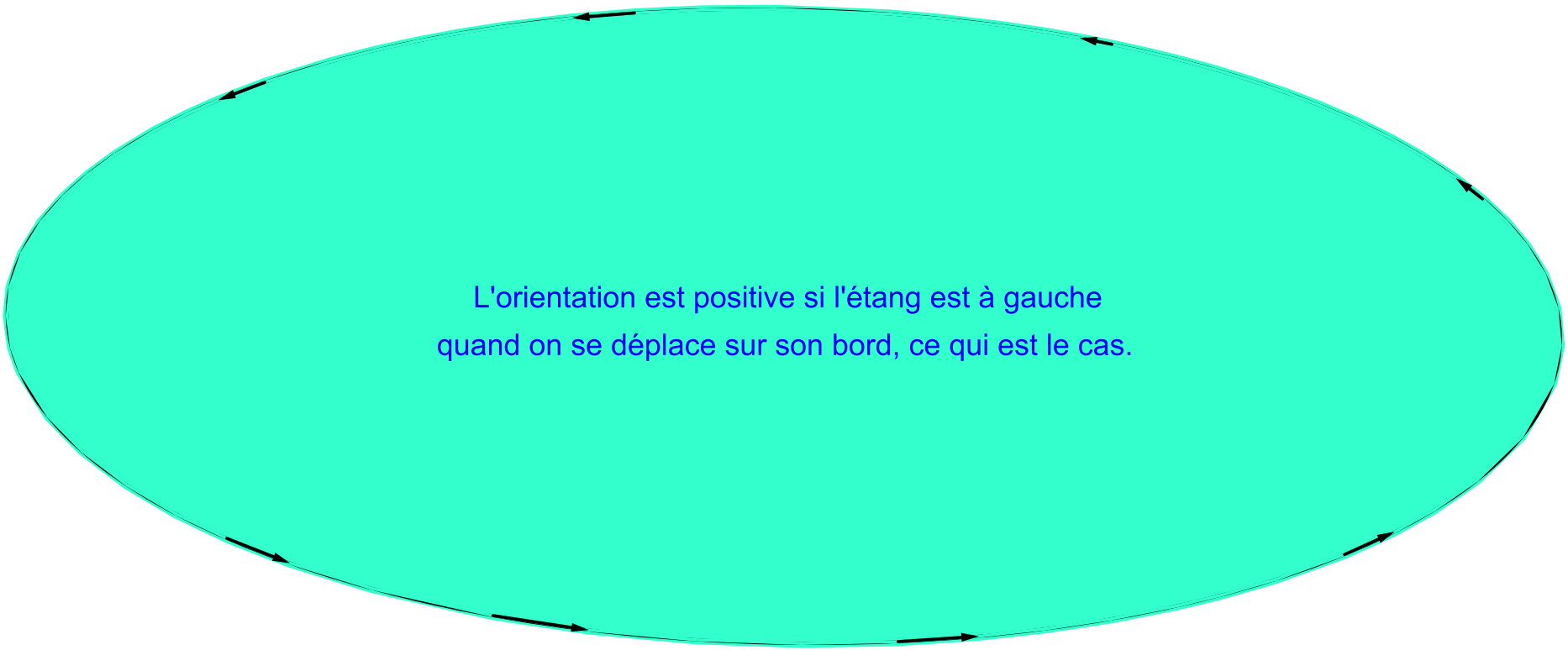


La notion d'orientation

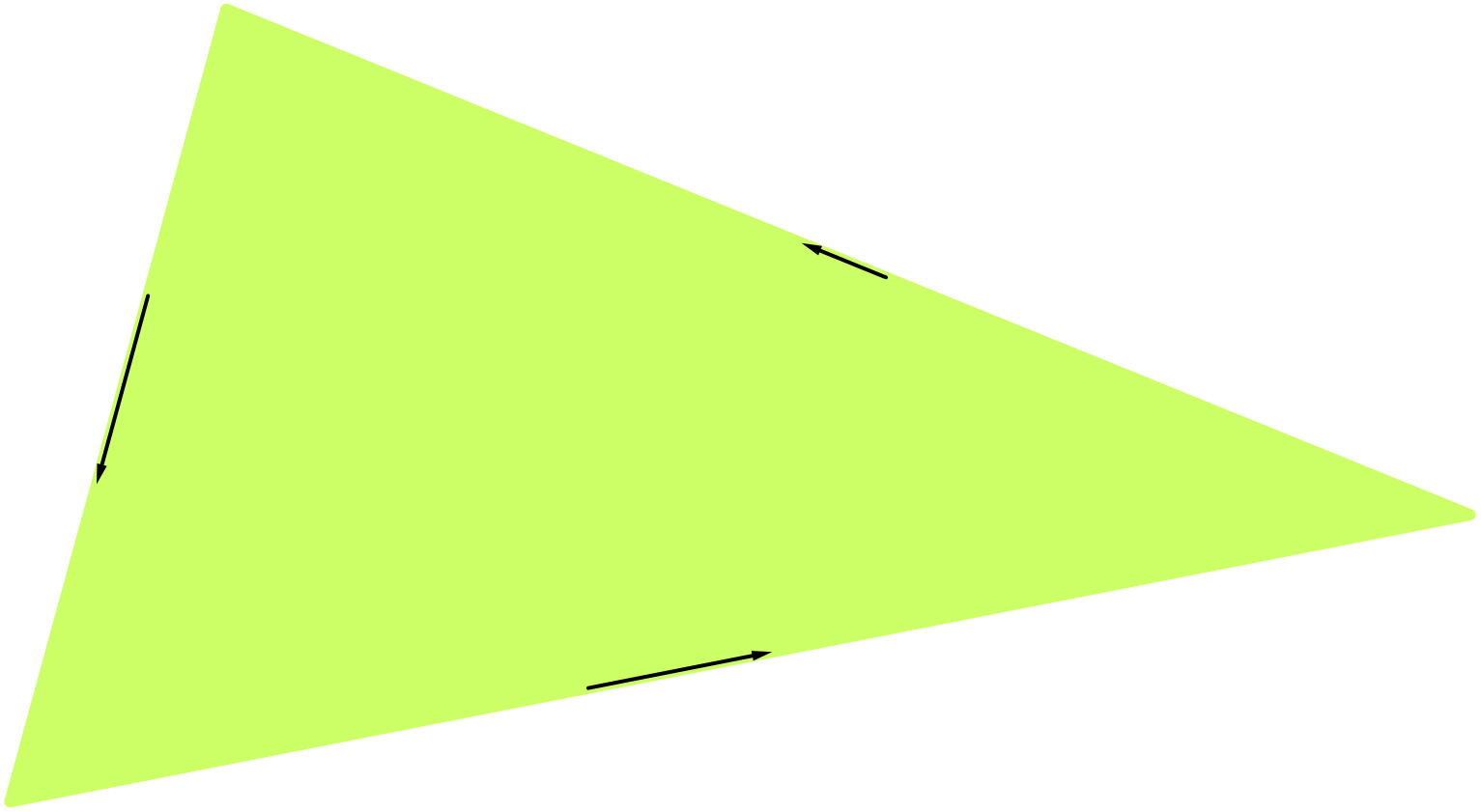


Une droite donnée définit une seule direction avec deux orientations

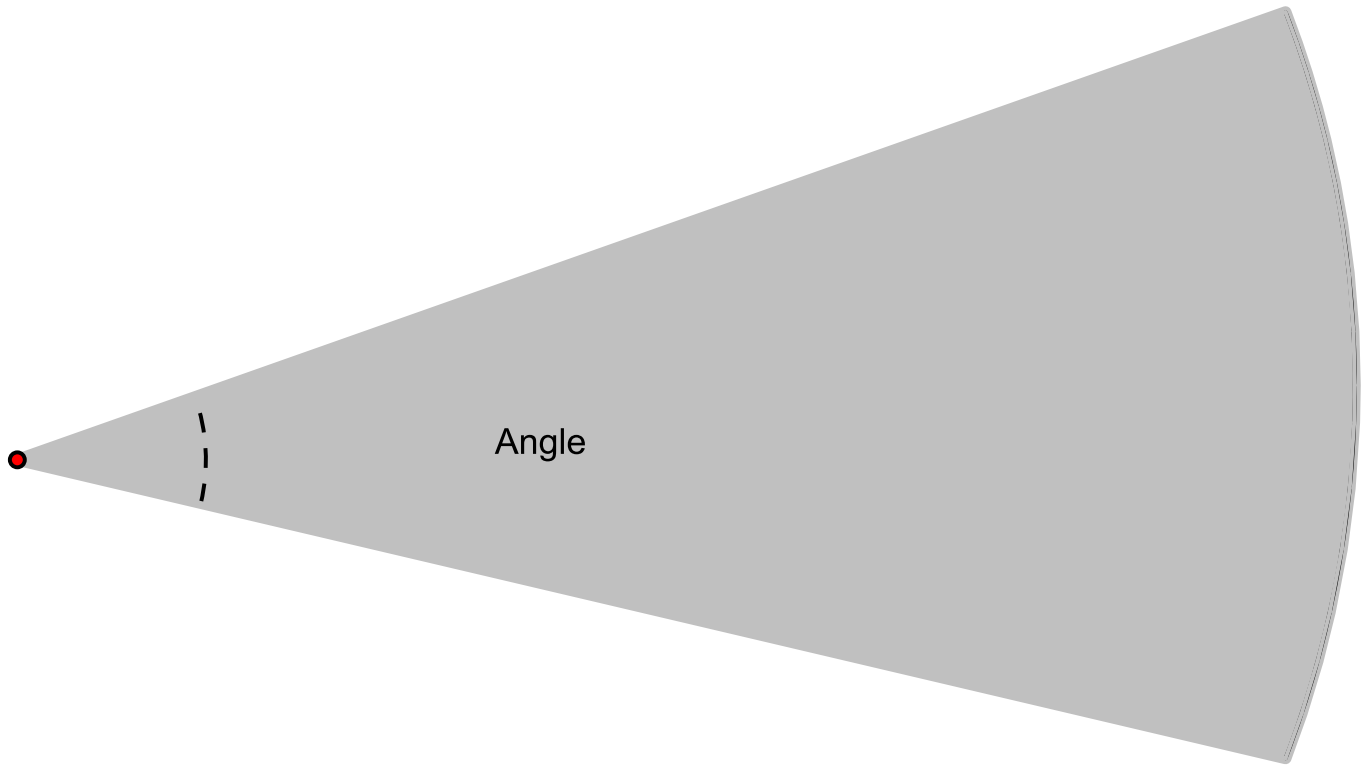
Mais comment orienter le plan ?



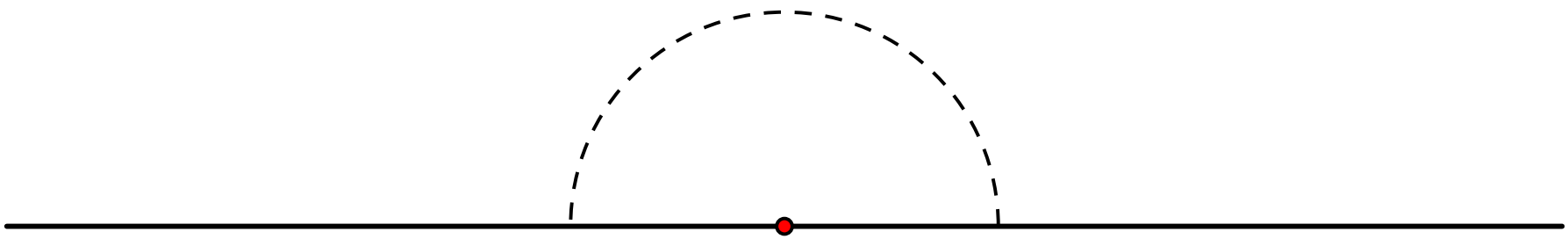
En particulier, voici un triangle orienté positivement !



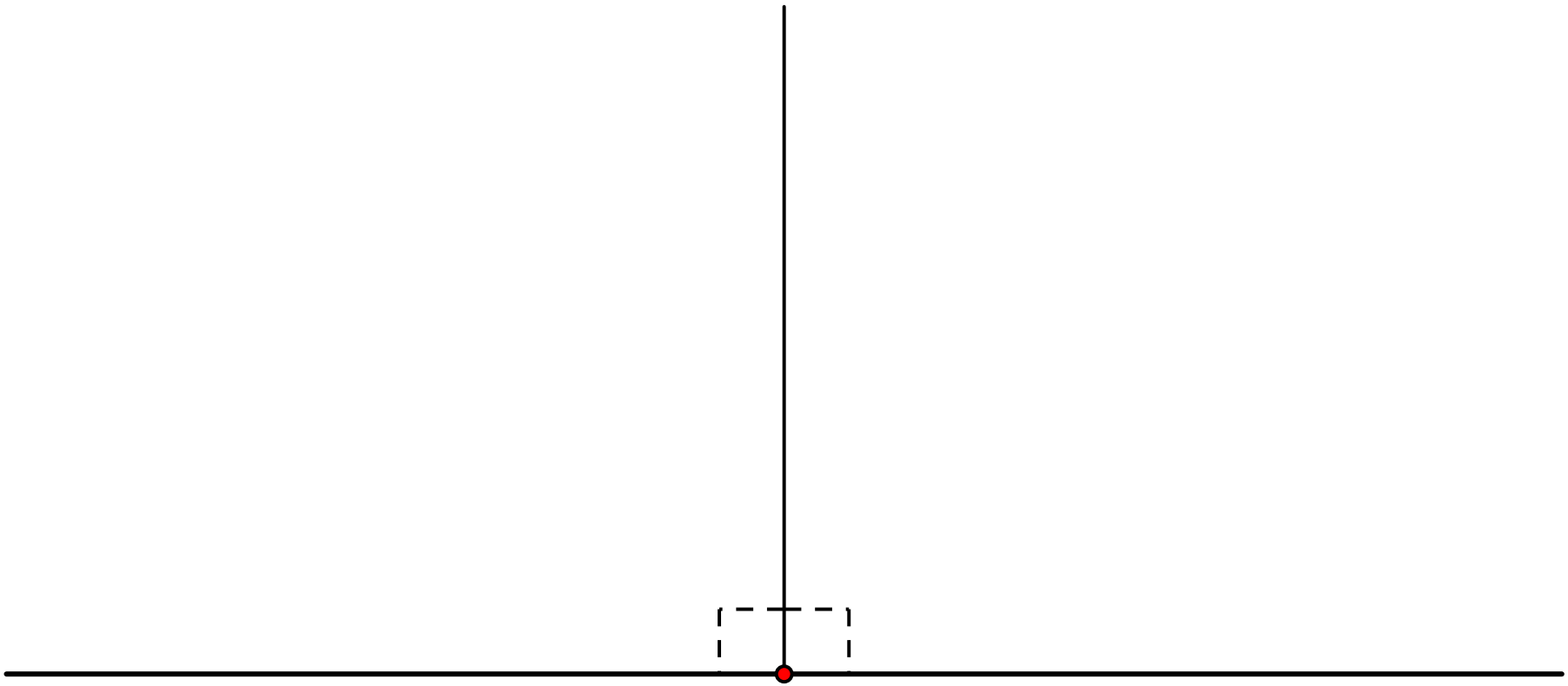
La notion d'angle



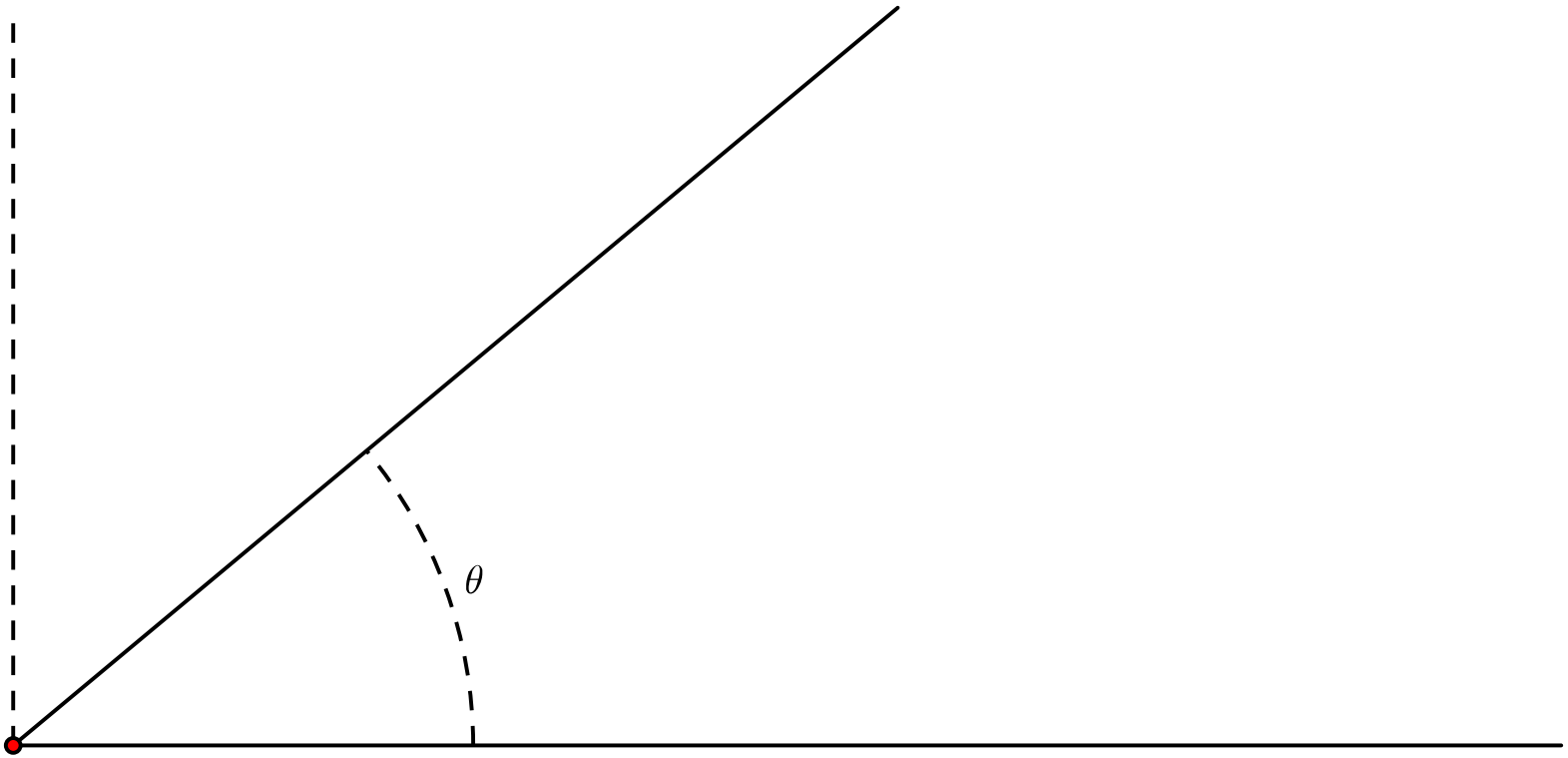
Angle plat



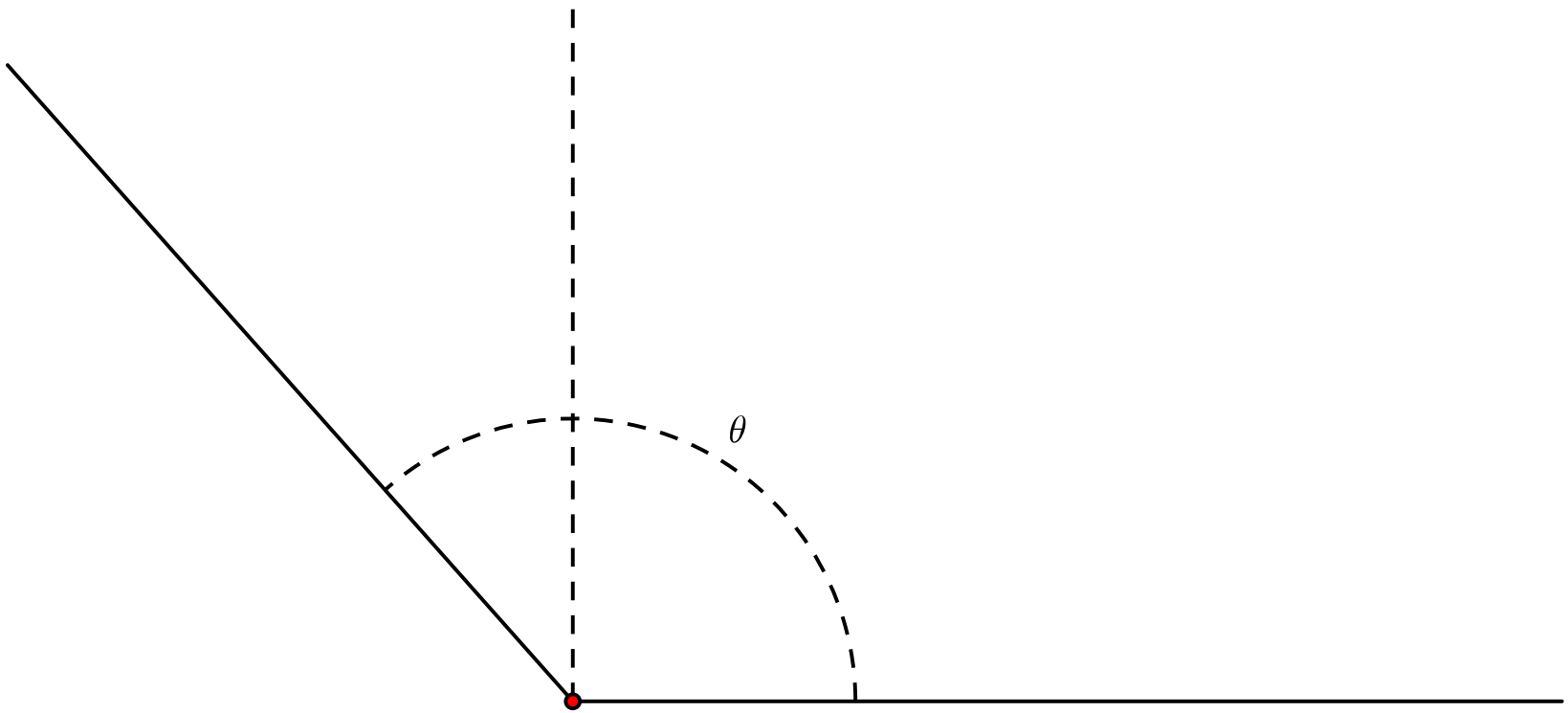
Angle droit



Angle aigu



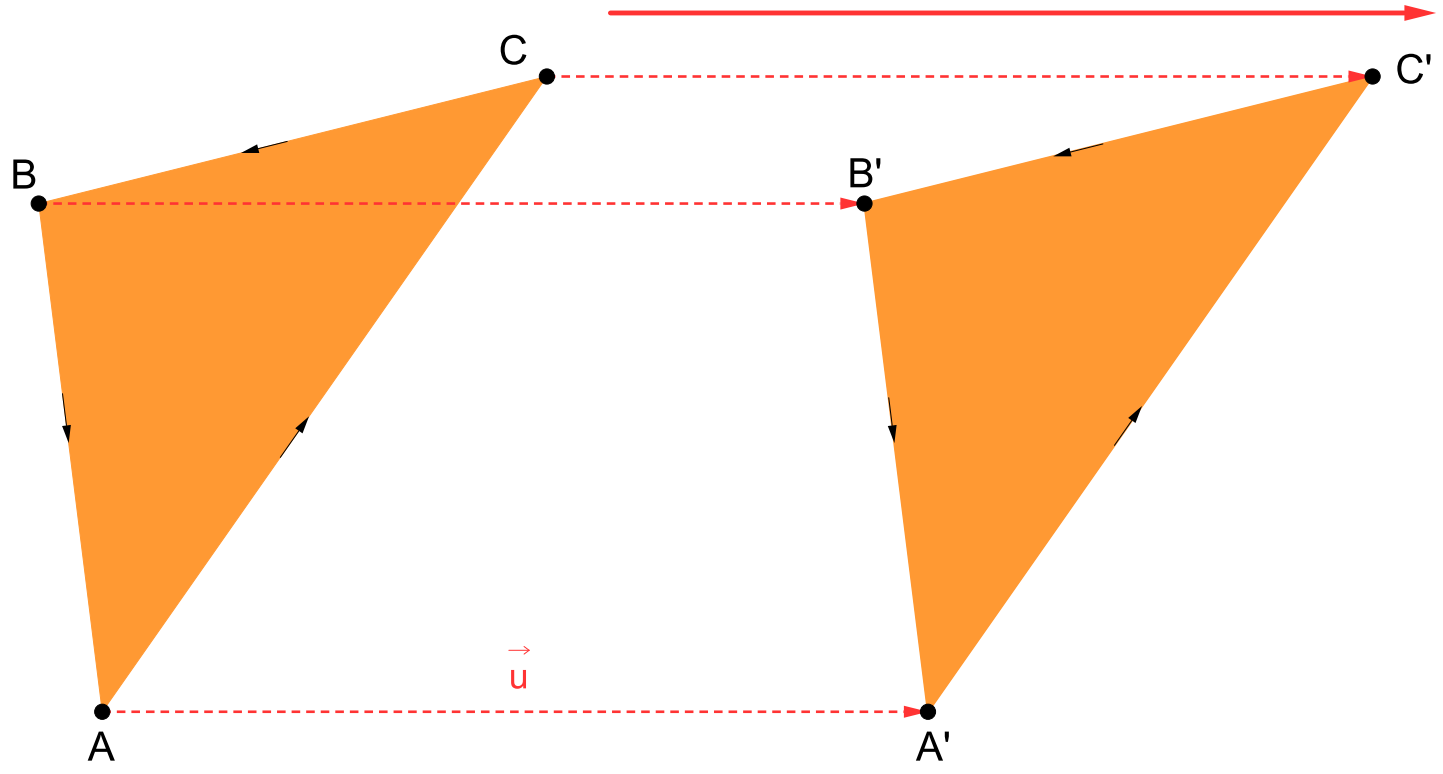
Angle obtu



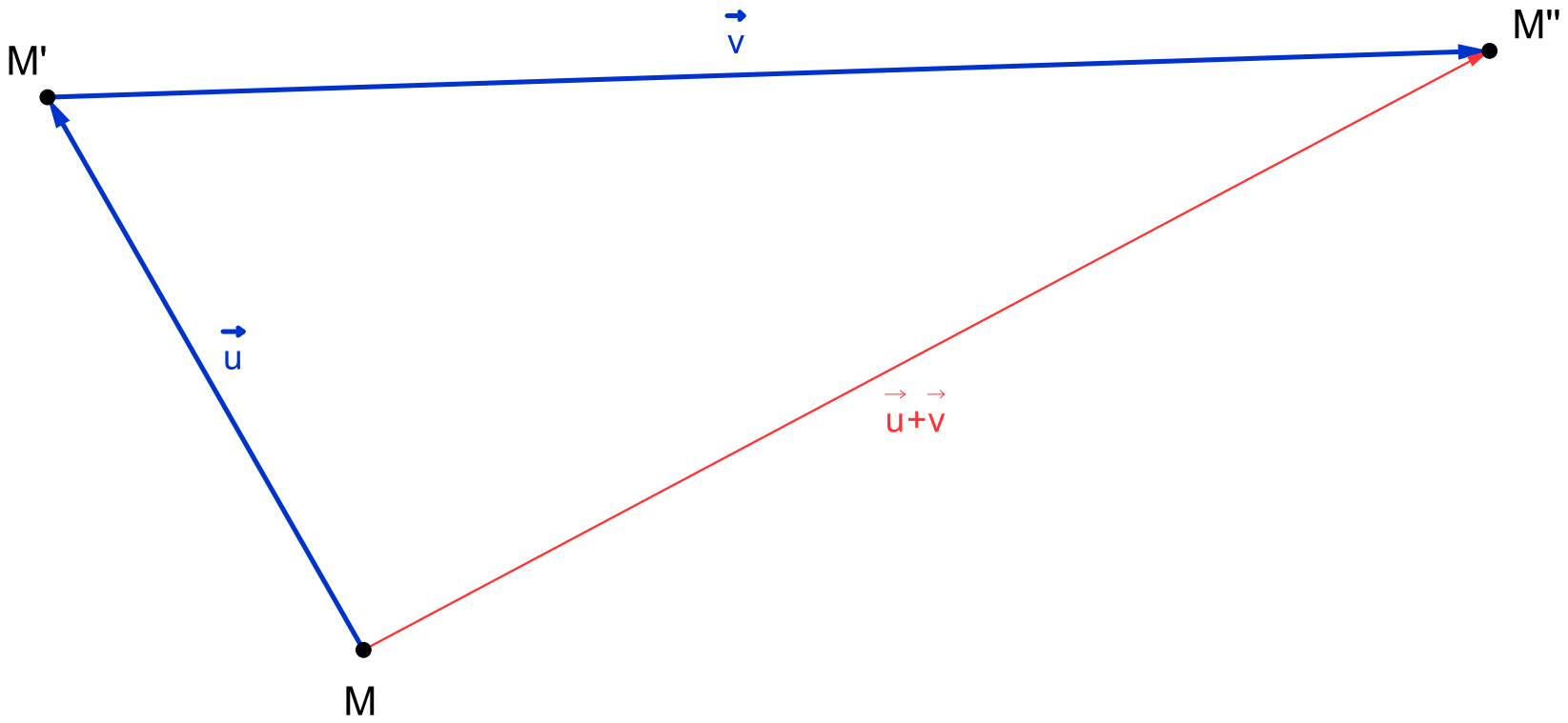
2. Mouvements euclidiens

Translation

Elle consiste à pousser les points par un vecteur $\overrightarrow{u}$. C'est un mouvement (ou transformation) qui conserve l'orientation.

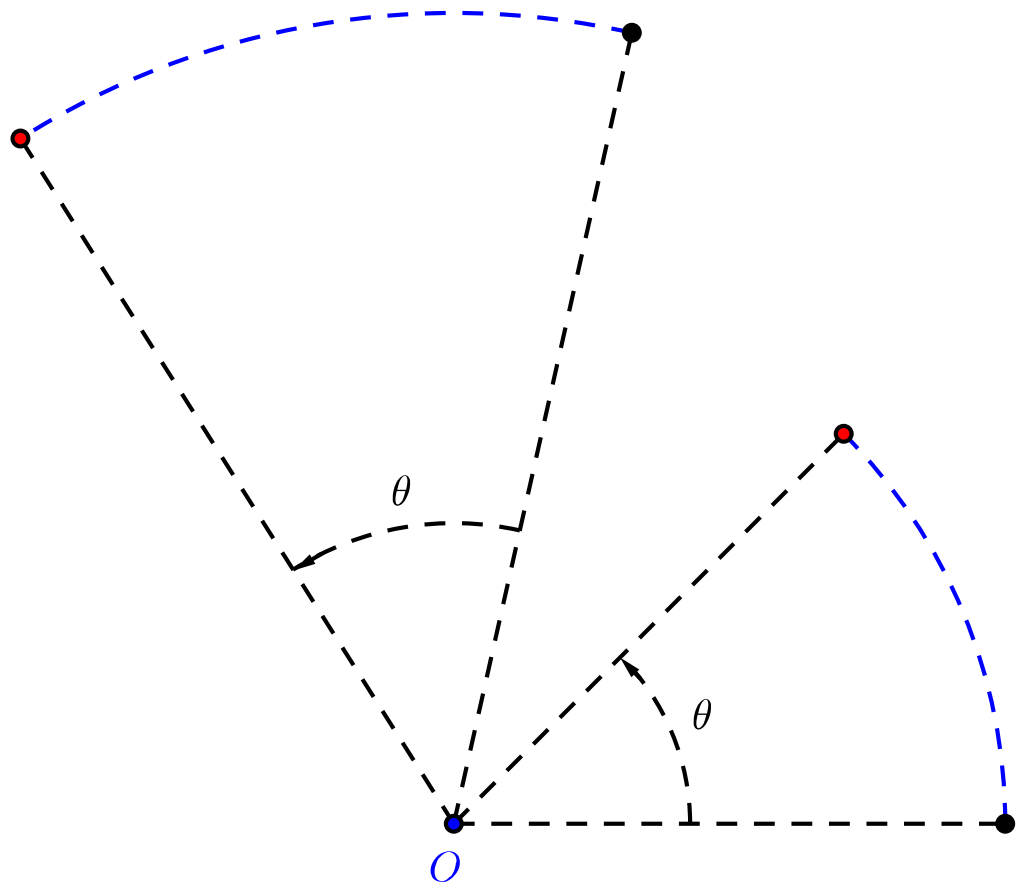


Les translations se composent comme les vecteurs.
La translation de vecteur nul est l'identité :
elle ne fait bouger aucun point.

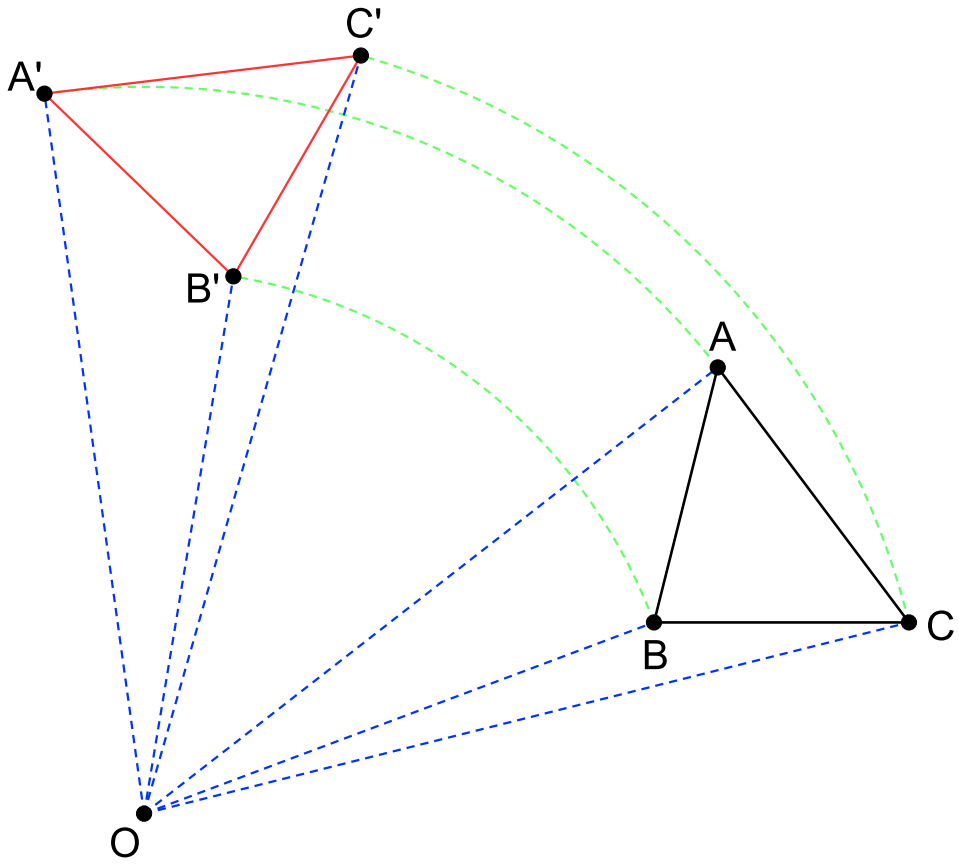


Rotation

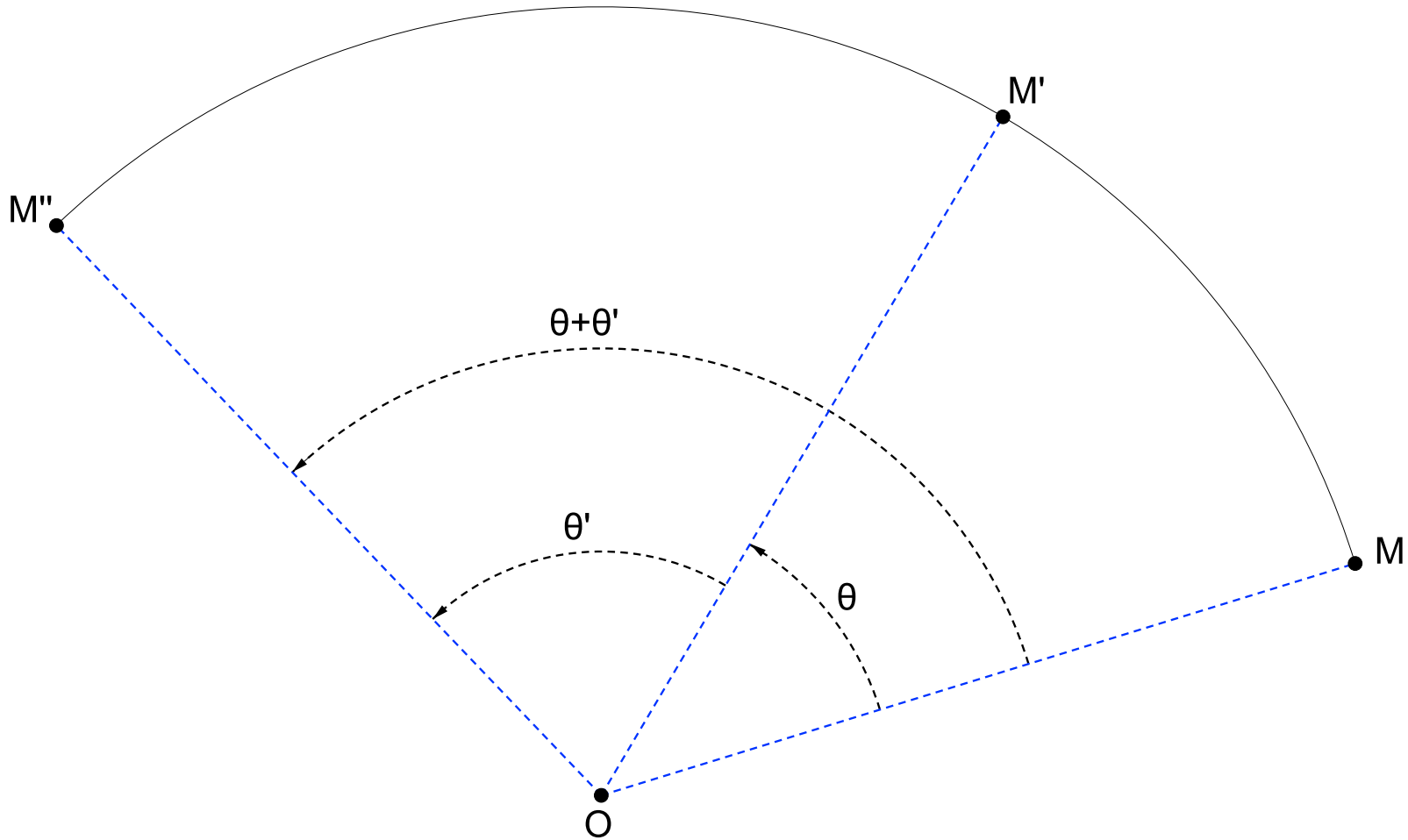
Elle consiste à faire tourner les points d'un angle θ appelé angle de rotation autour d'un point O appelé centre de rotation.

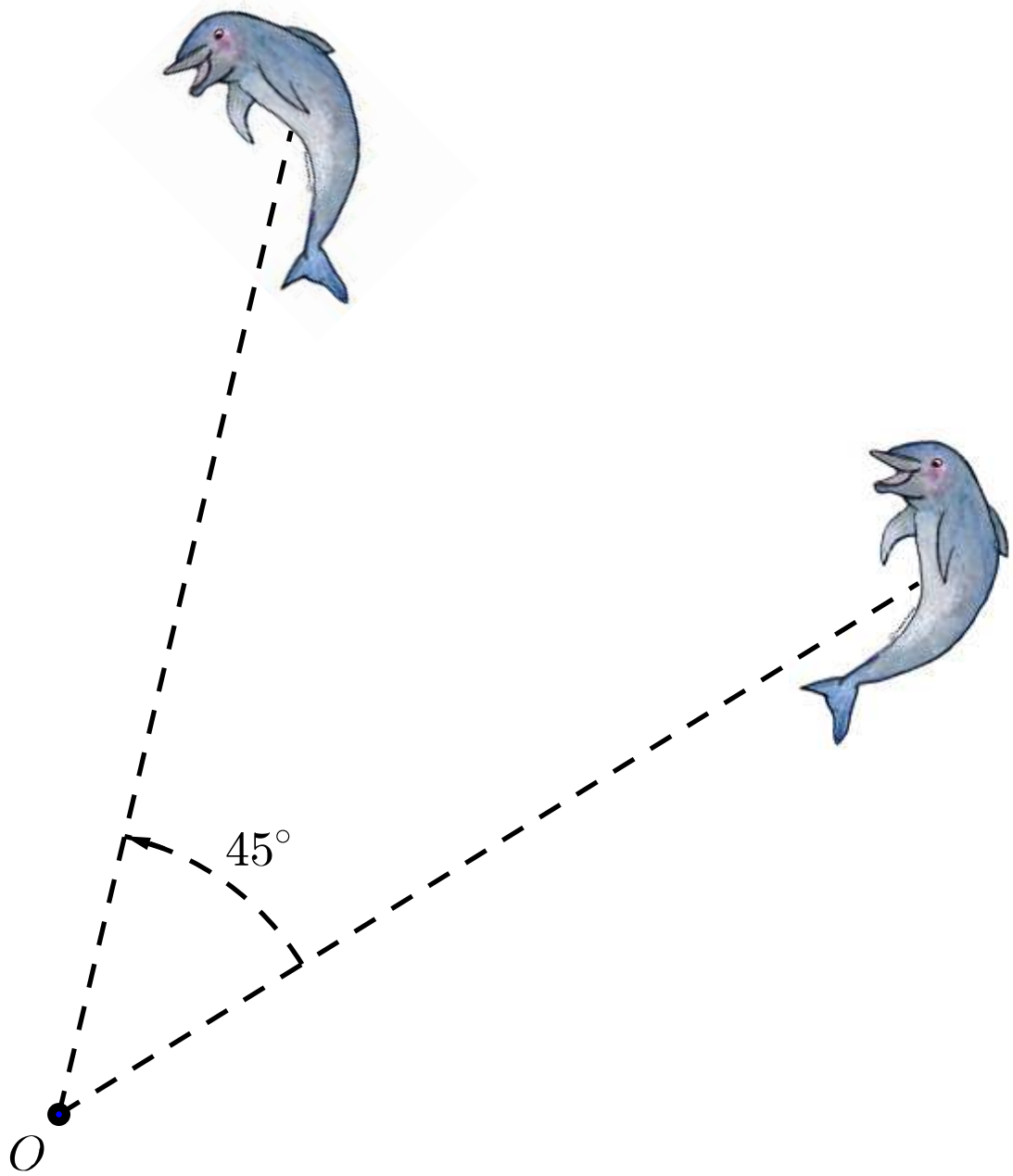


C'est un mouvement euclidien qui conserve l'orientation.



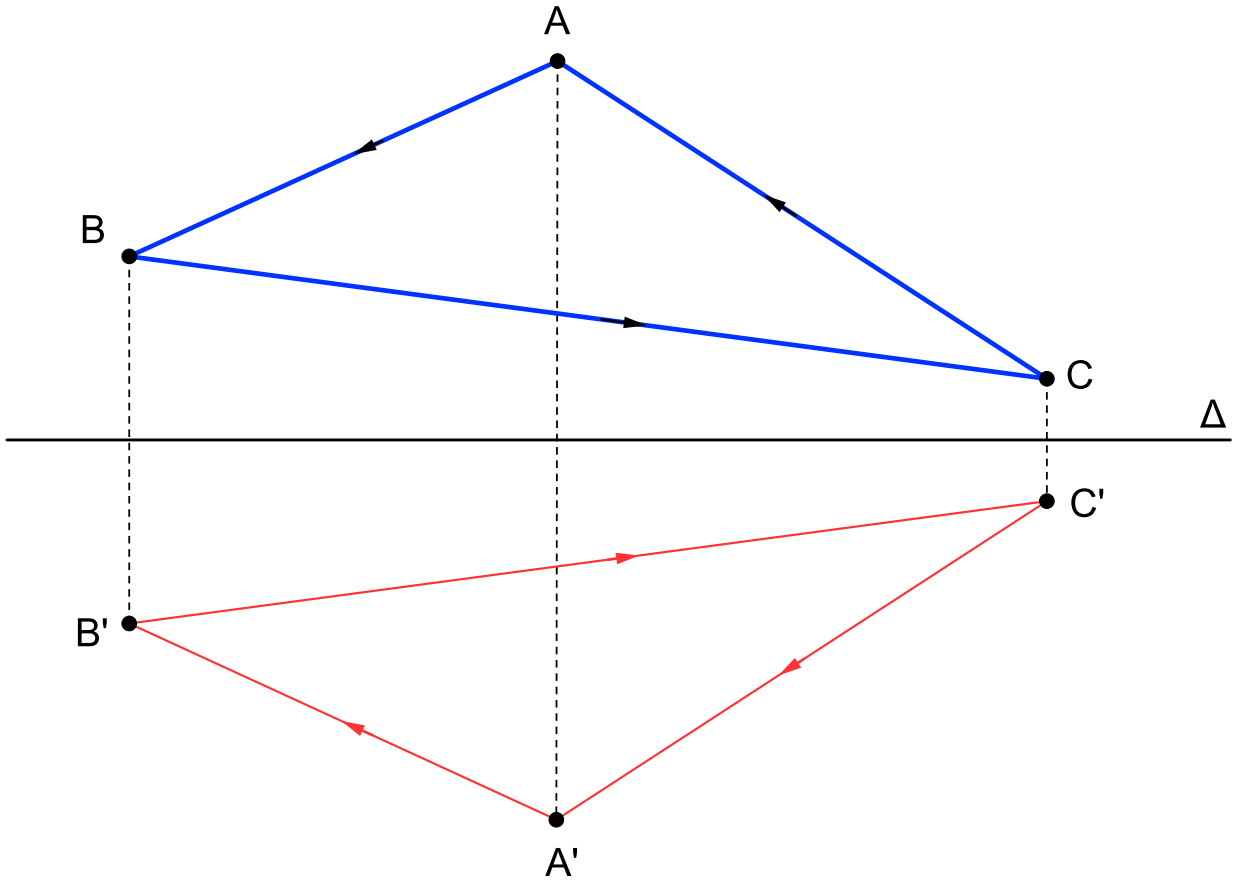
La composée de deux rotations de centre O et d'angles respectifs θ et θ' est une rotation de même centre O et d'angle $\theta + \theta'$.



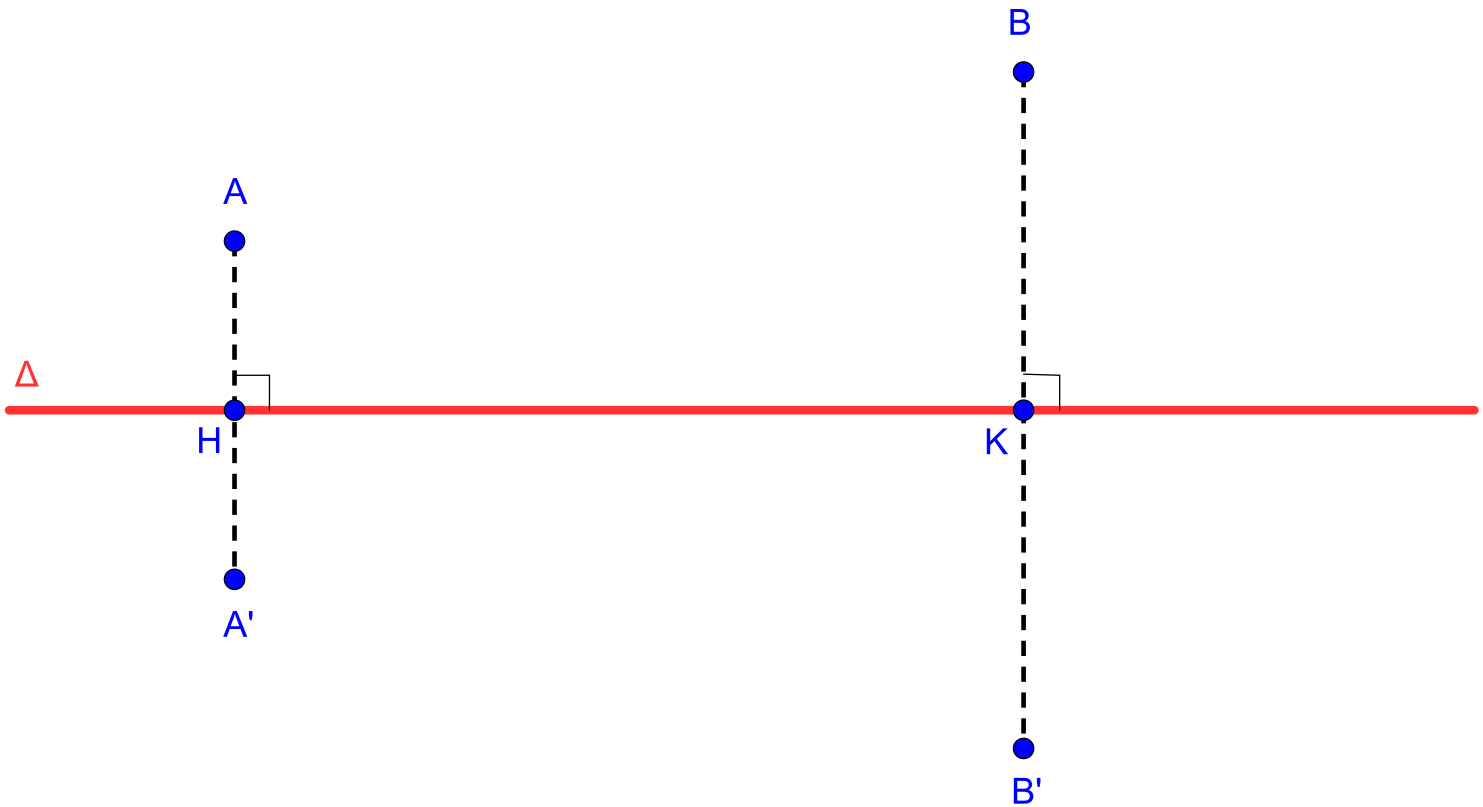

45°
O

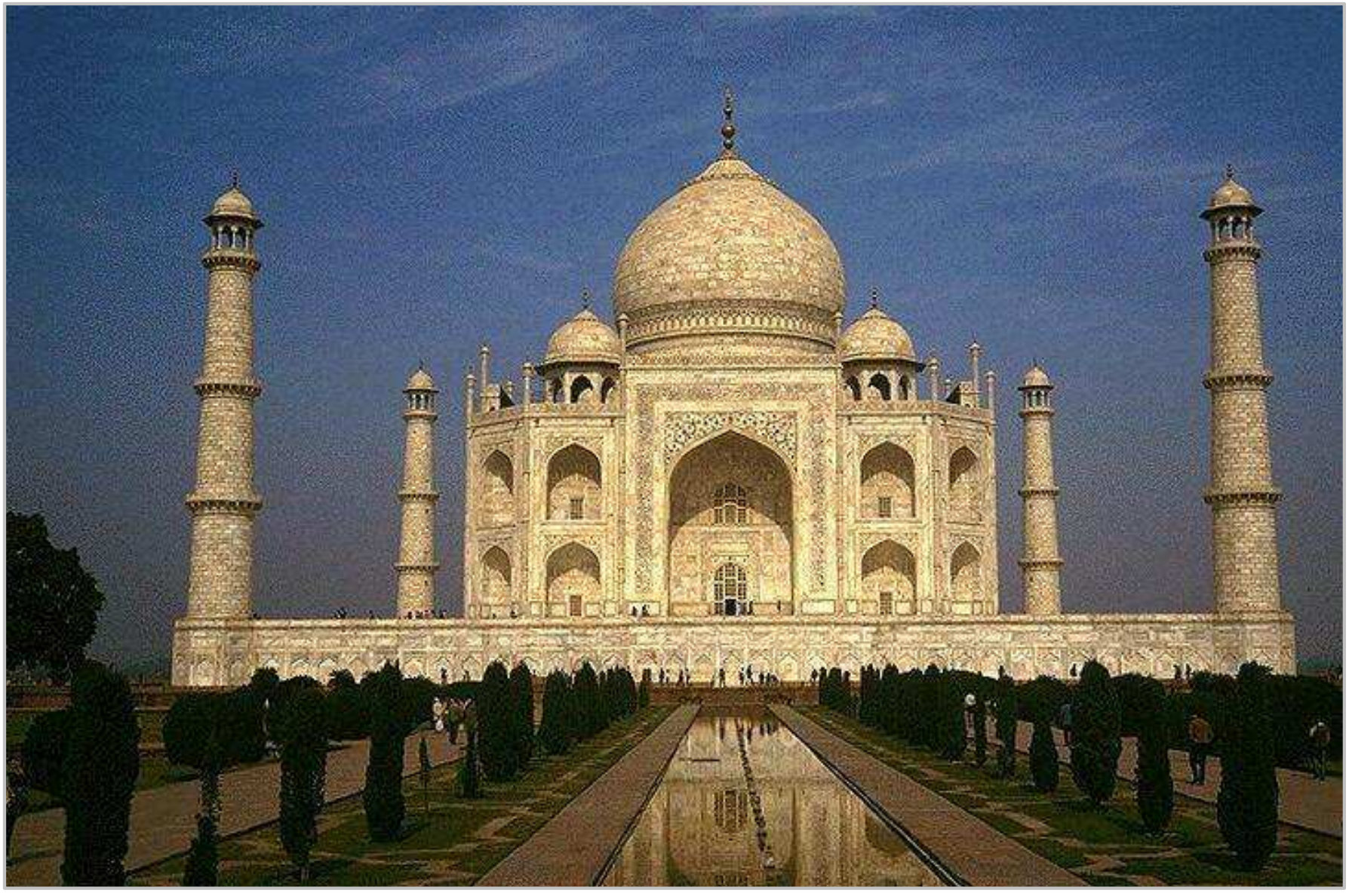
Réflexion ou symétrie axiale

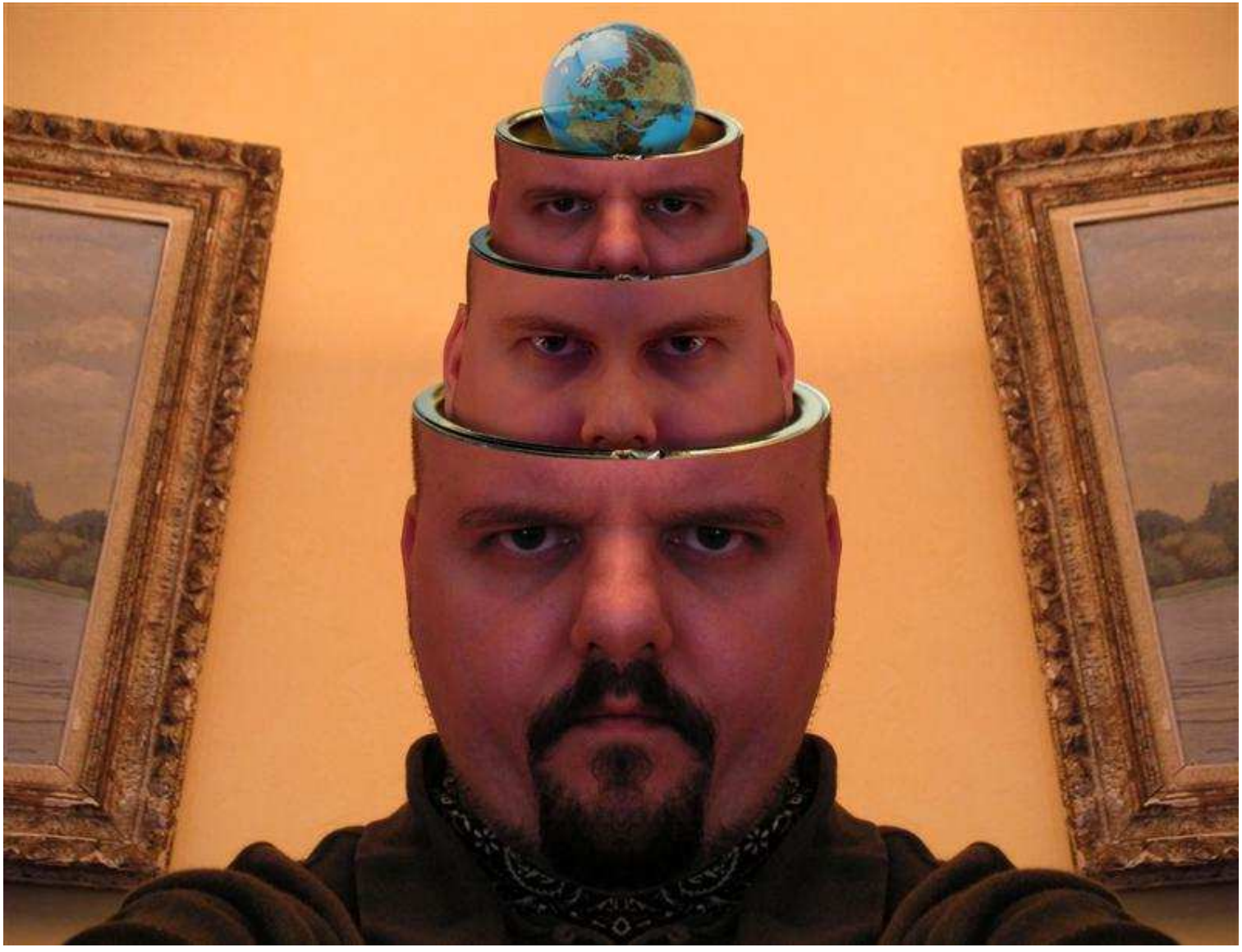
C'est regarder l'image d'un objet dans un miroir ! Elle est donc associée à une droite appelée axe de réflexion (ou axe de symétrie). Elle ne conserve pas l'orientation.

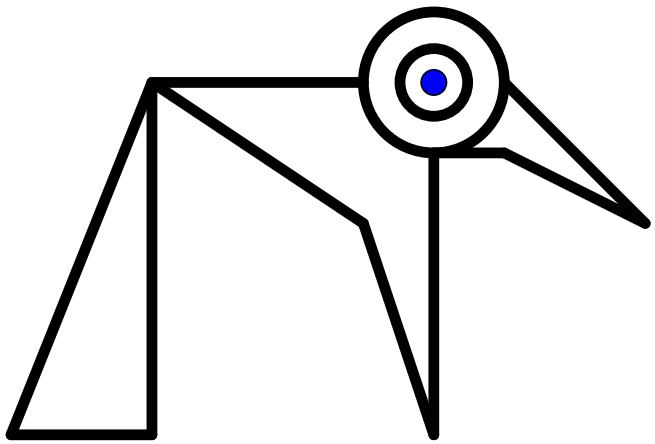


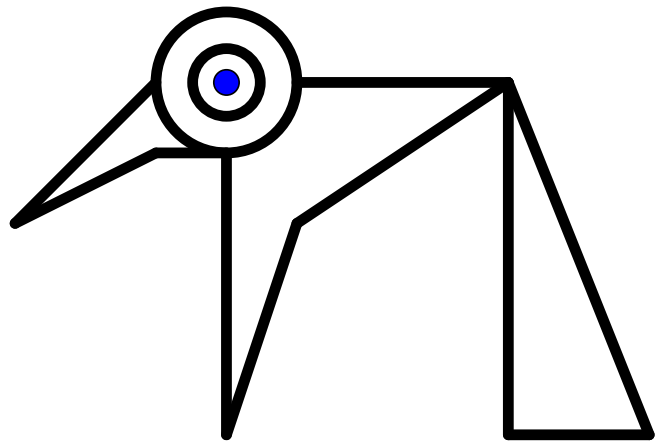
- Une symétrie axiale est définie par une droite Δ.
- À tout point A correspond un point A' tel que :
  1. $A = A'$ si A est sur Δ, sinon
  2. la droite (AA') est perpendiculaire à Δ en un point H et on a l'égalité des longueurs $AH = HA'$.
- On dira que Δ est l'axe de la symétrie.

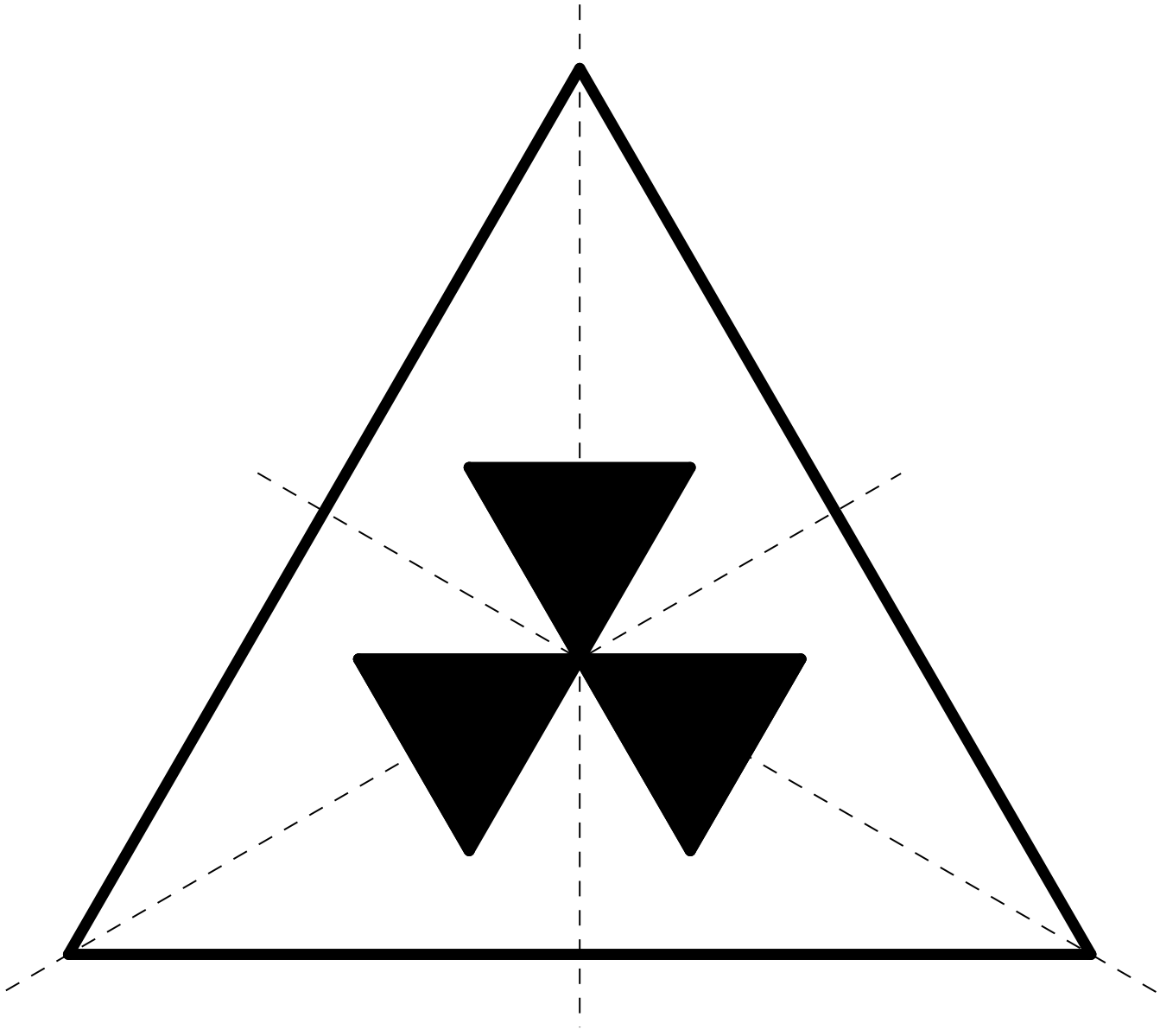




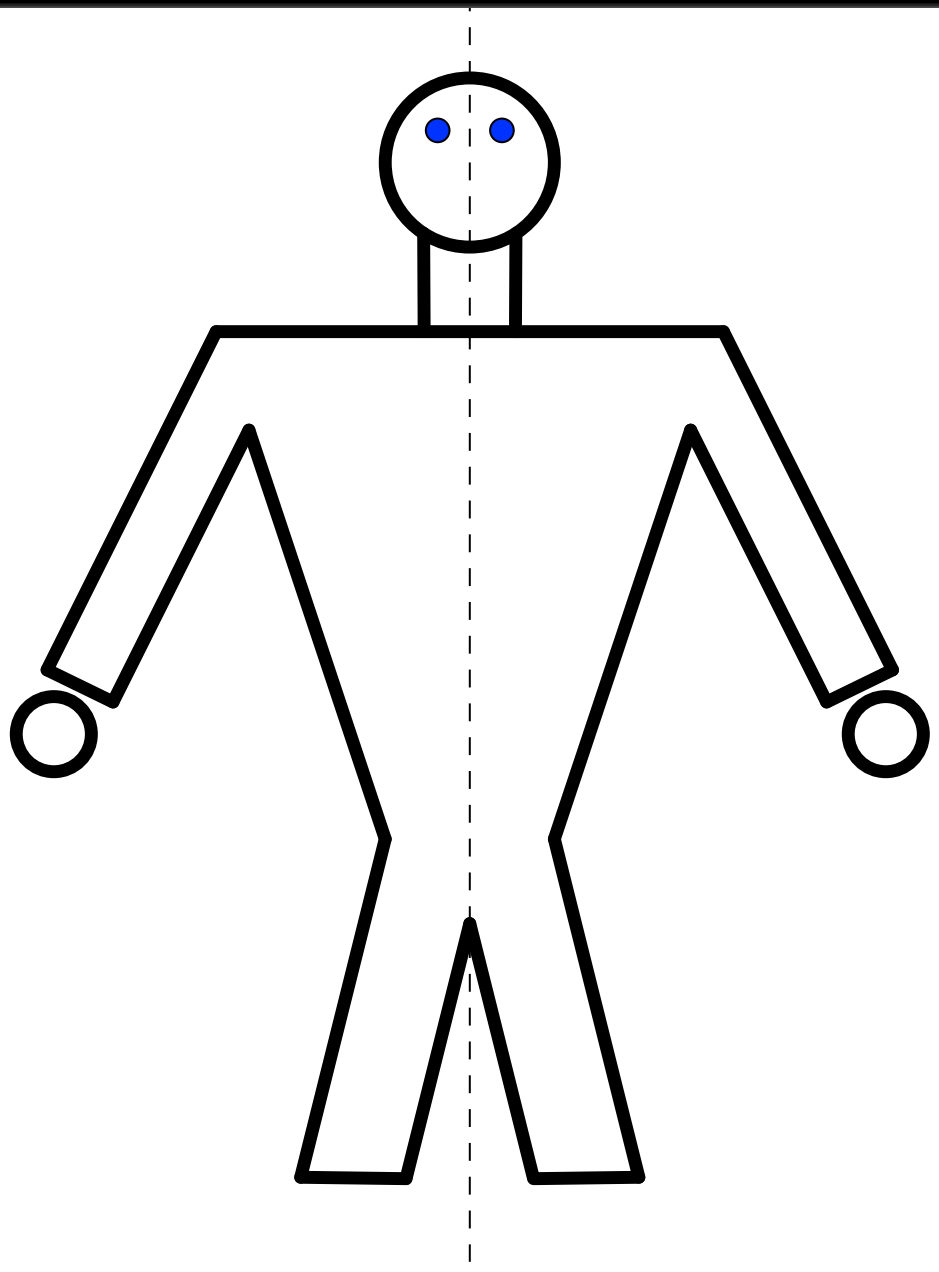




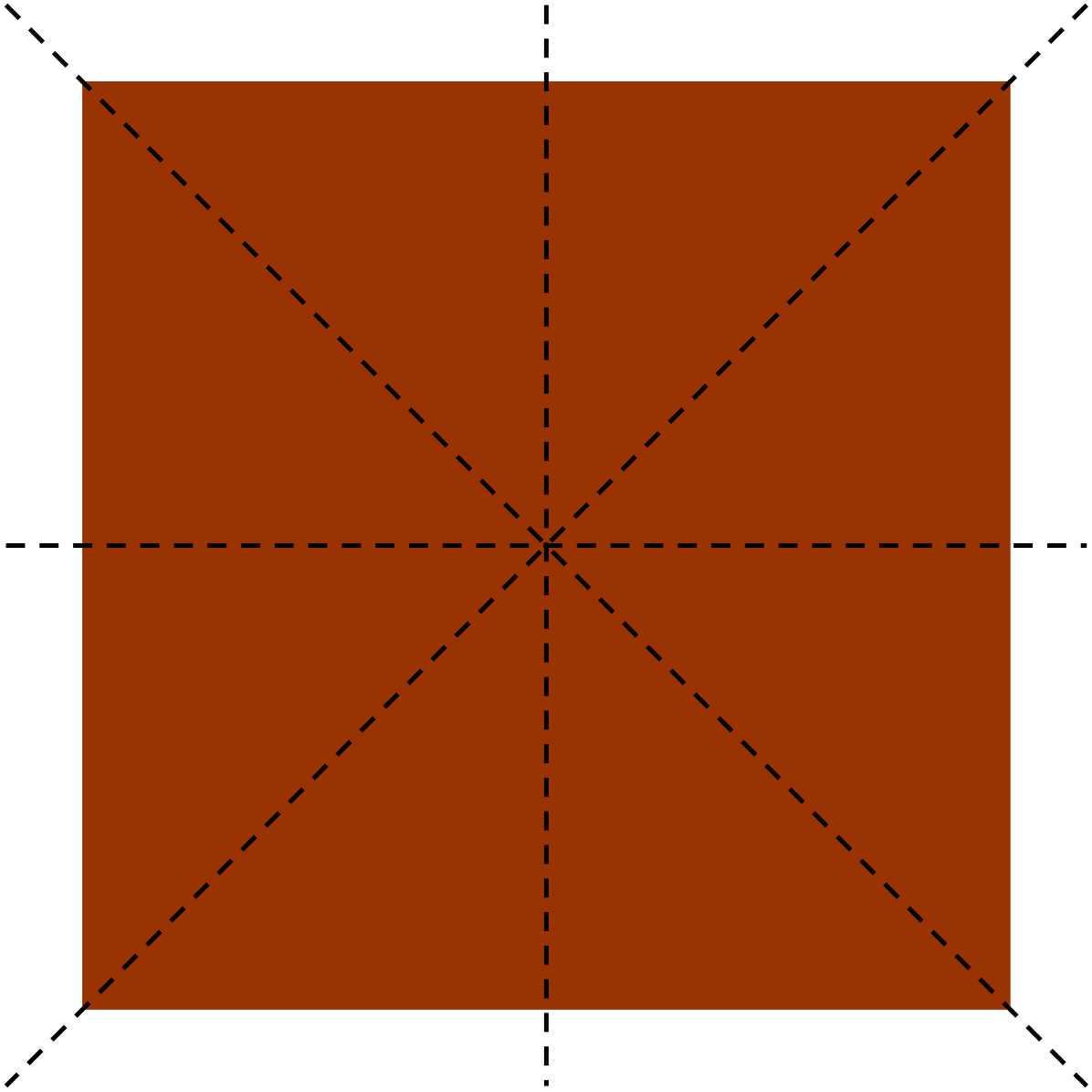


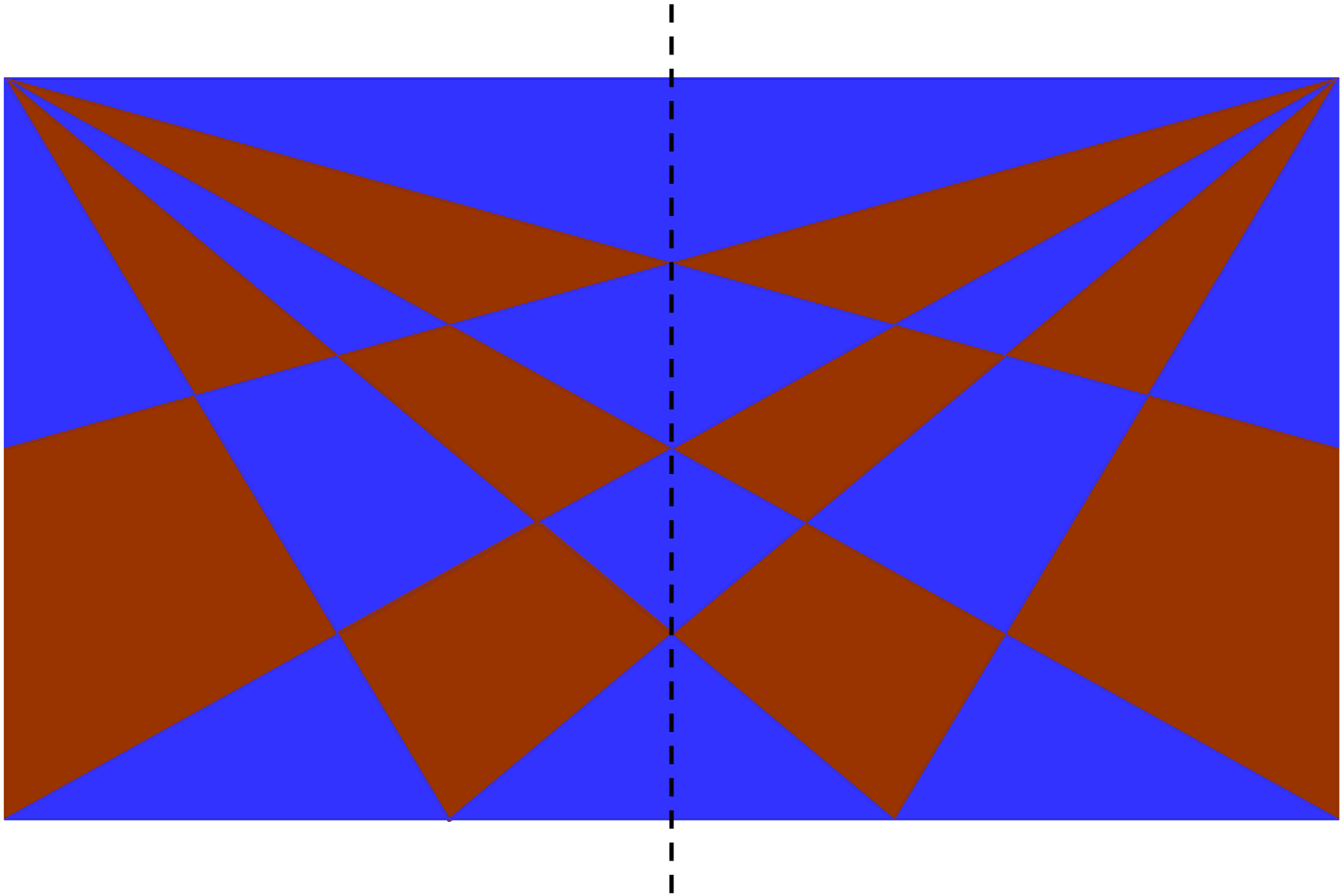


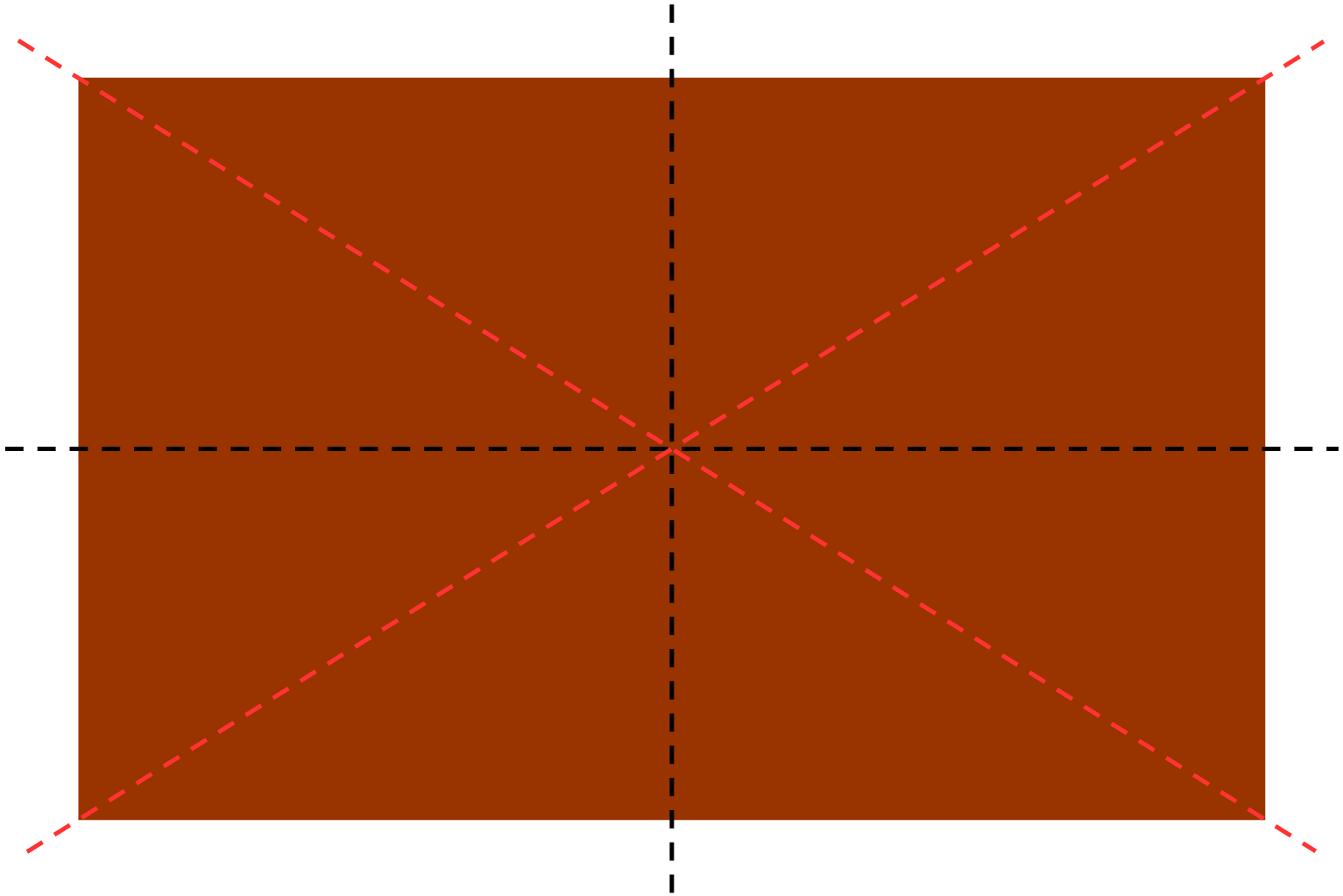
Attention : risque d'irradiation !

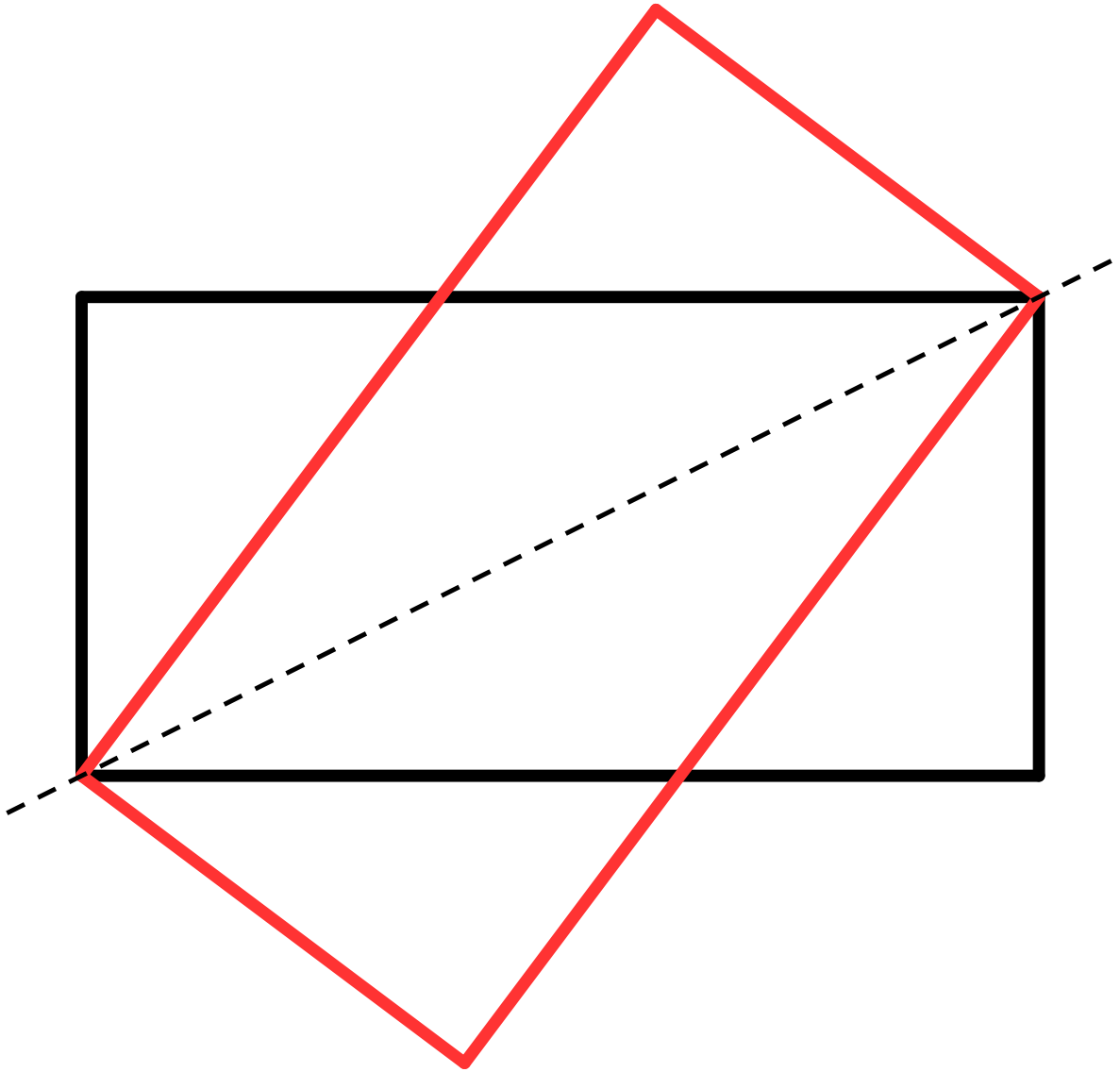


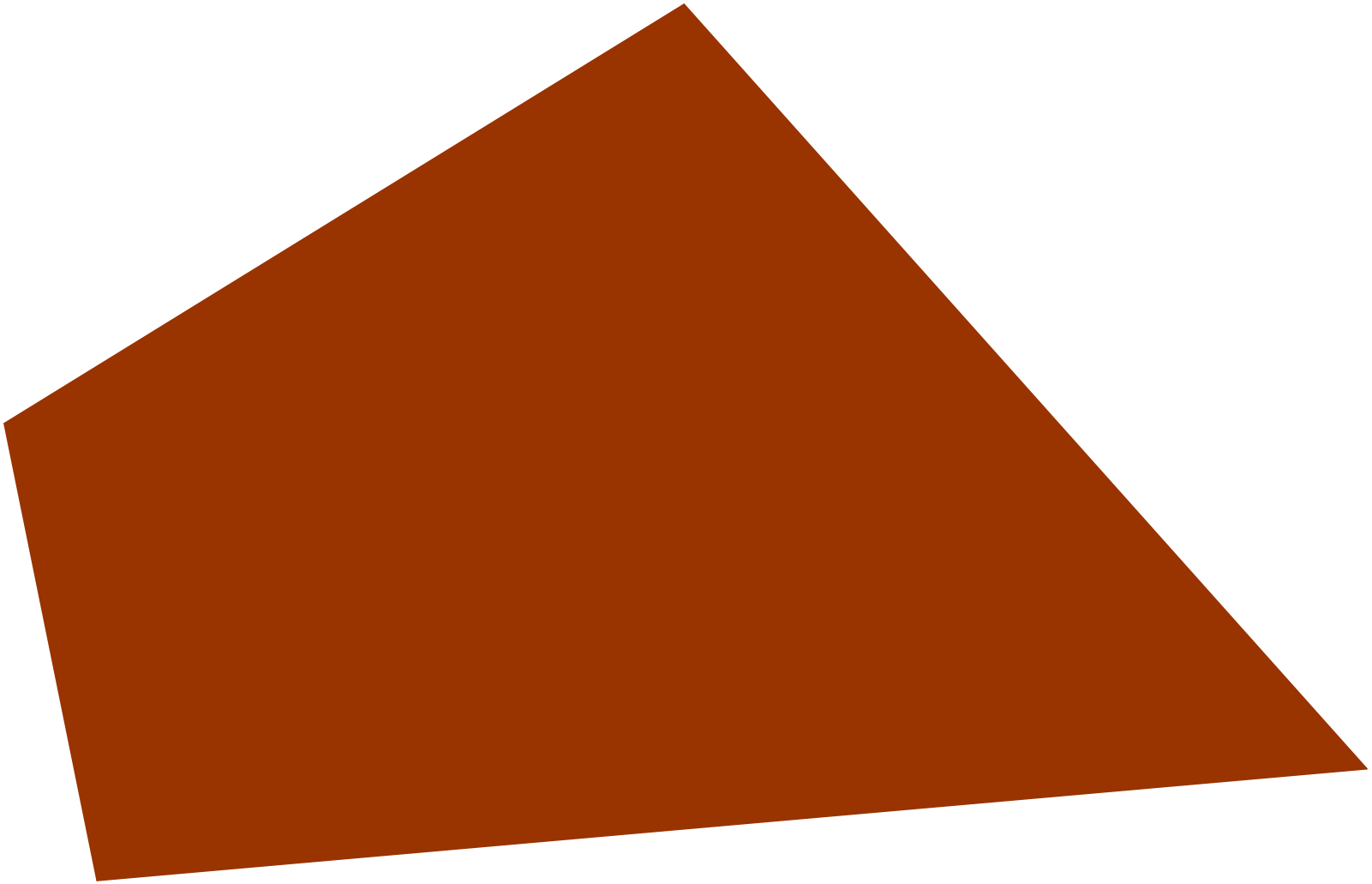
Premier étage à droite !

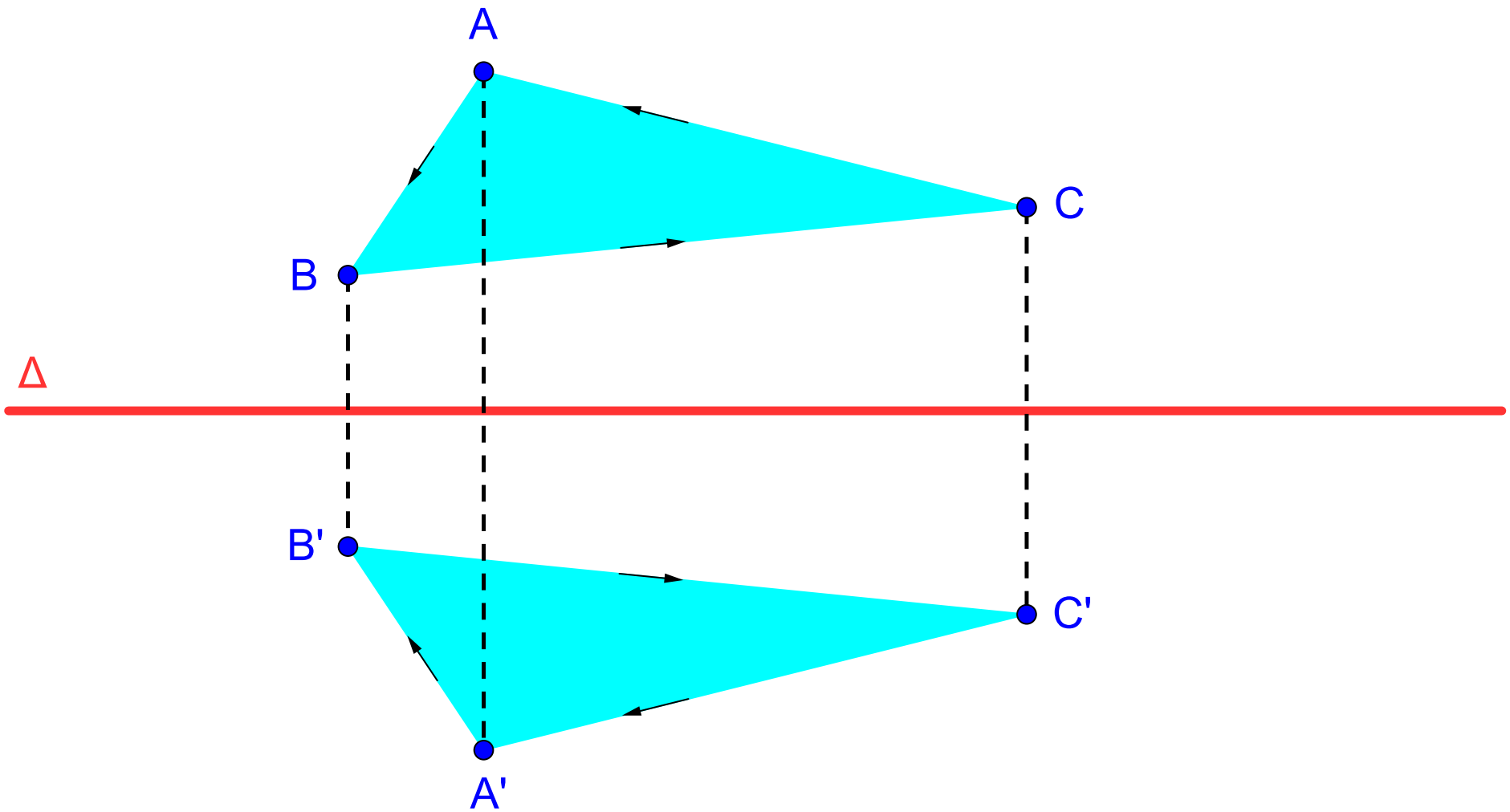


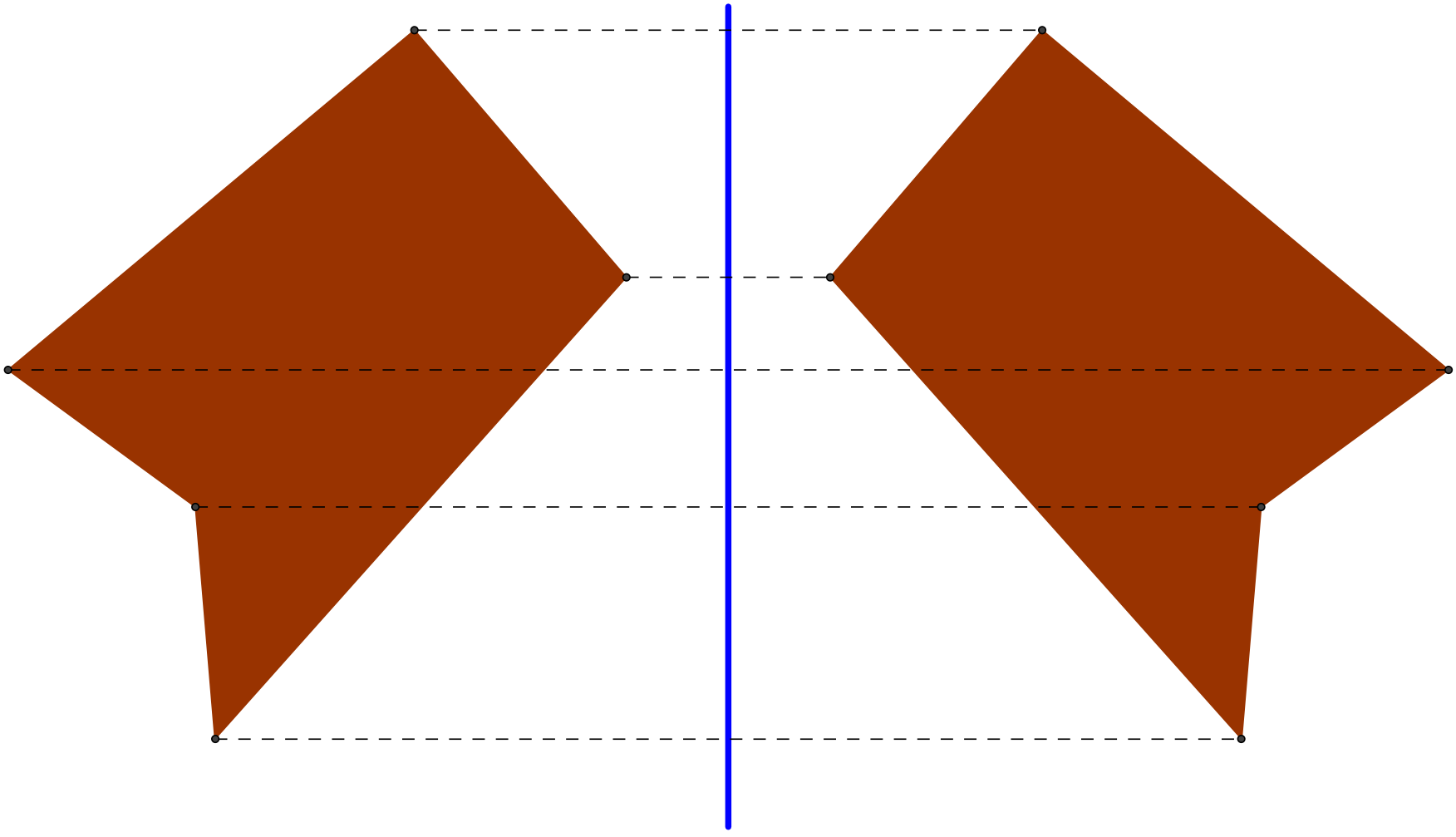


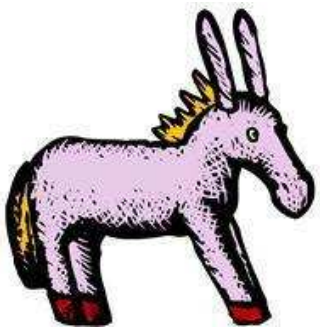


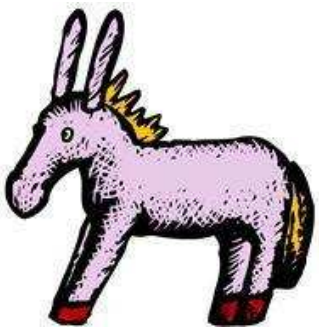




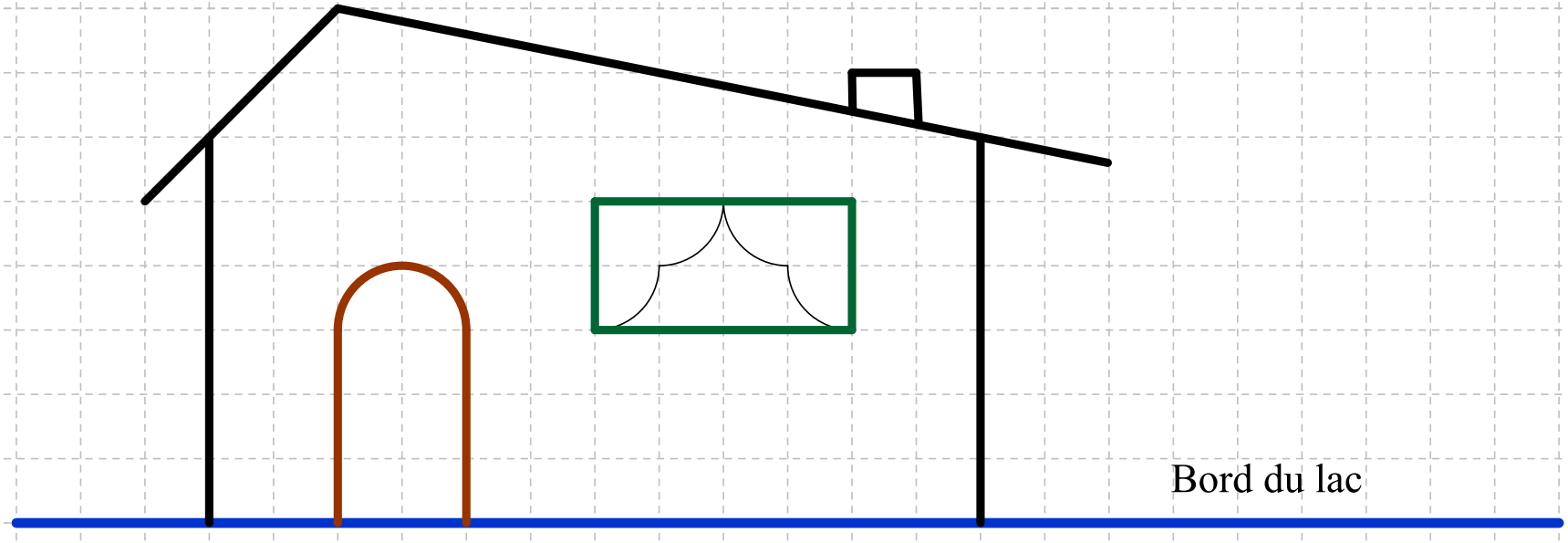

A
B
C
Δ
B'
C'
A'



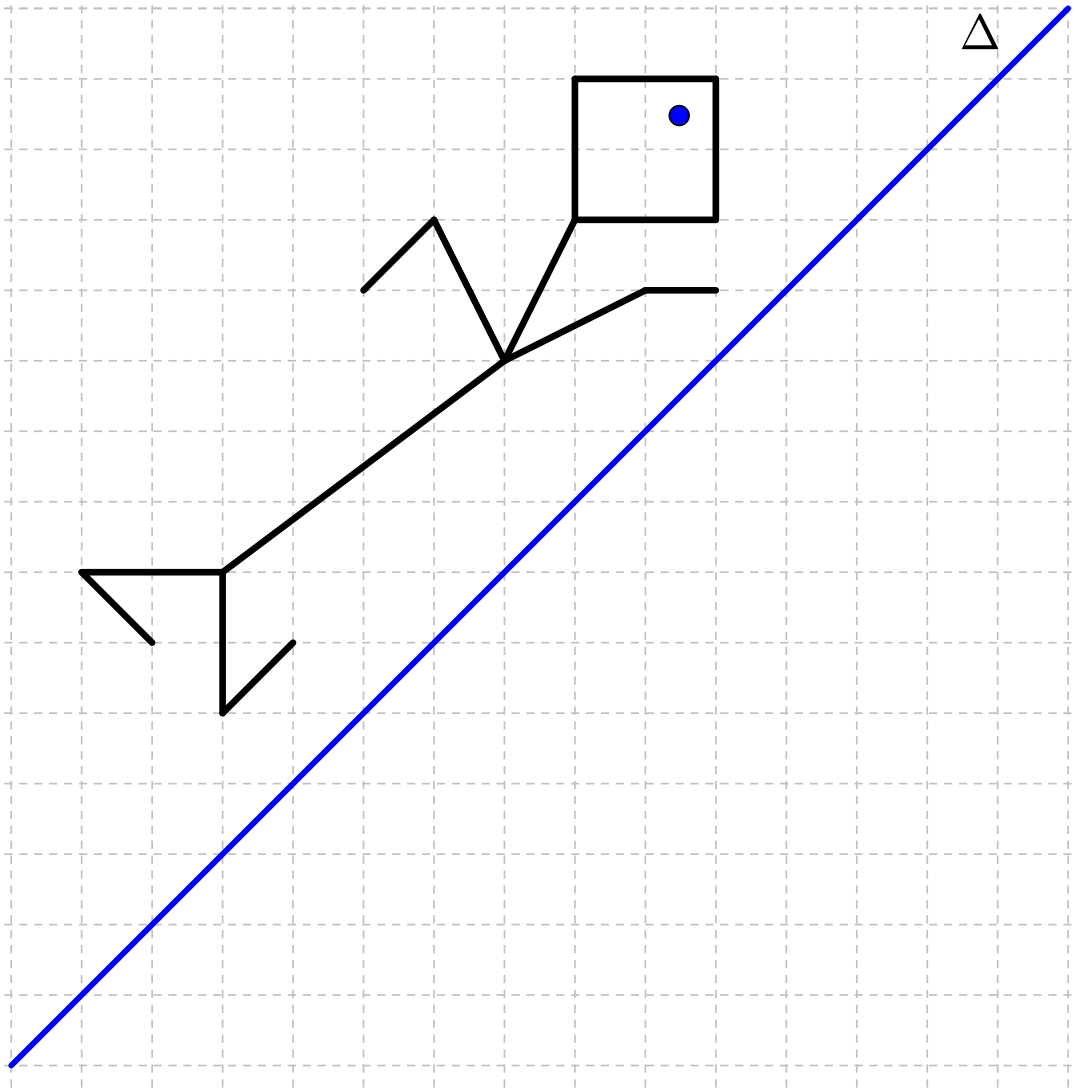




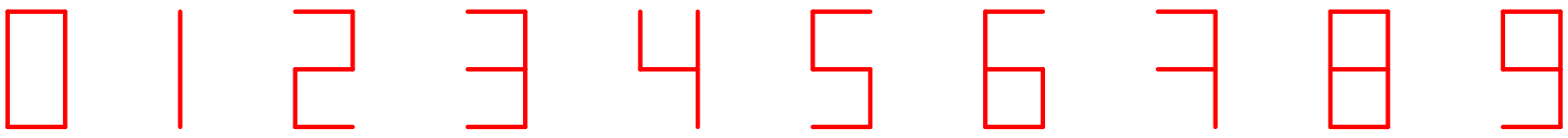
Cette maison est située au bord d'un lac.
Dessiner son reflet dans l'eau.

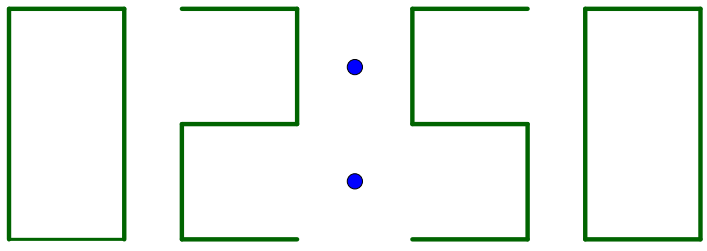


En utilisant uniquement une règle non graduée, tracer le symétrique du bonhomme par rapport à la droite.



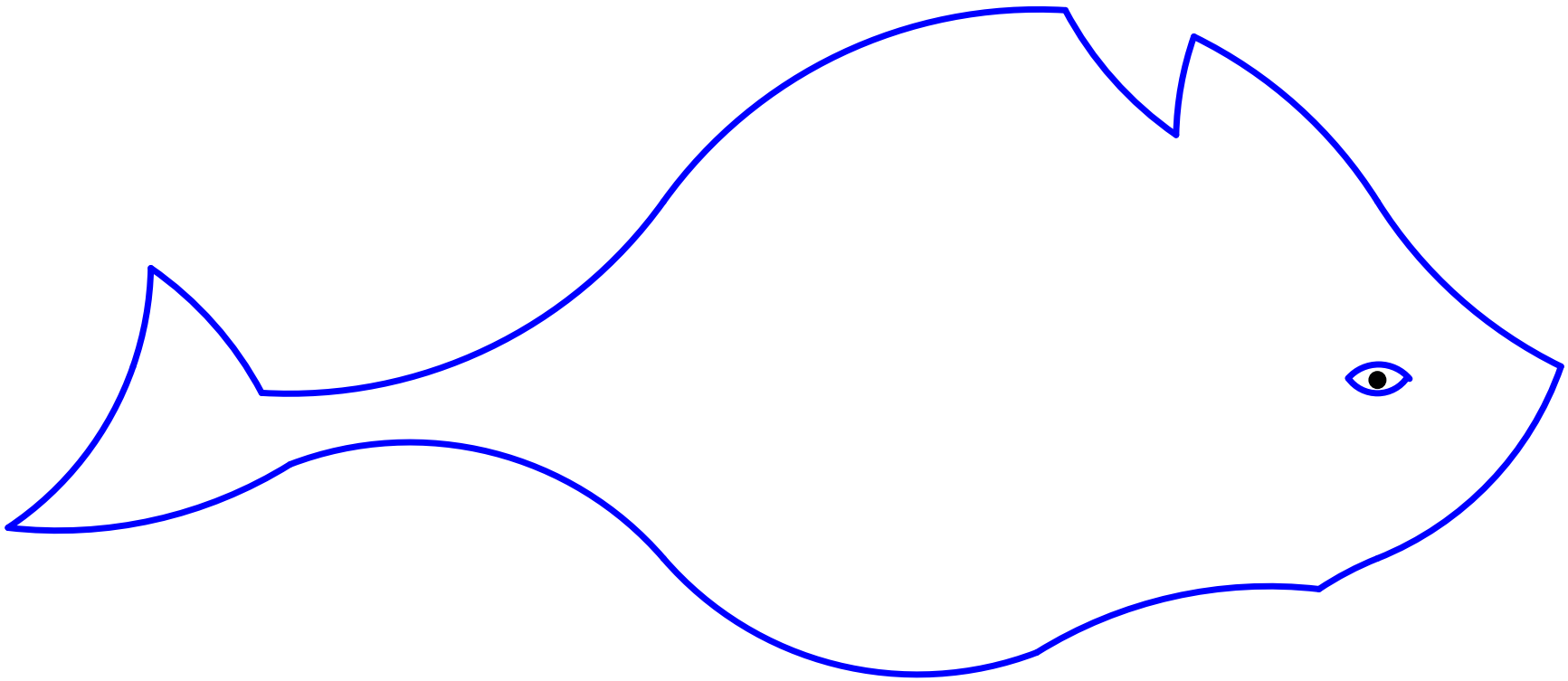
Donner tous les chiffres qui admettent un axe de symétrie (vertical, horizontal...), un centre de symétrie ainsi que toutes les heures symétriques.

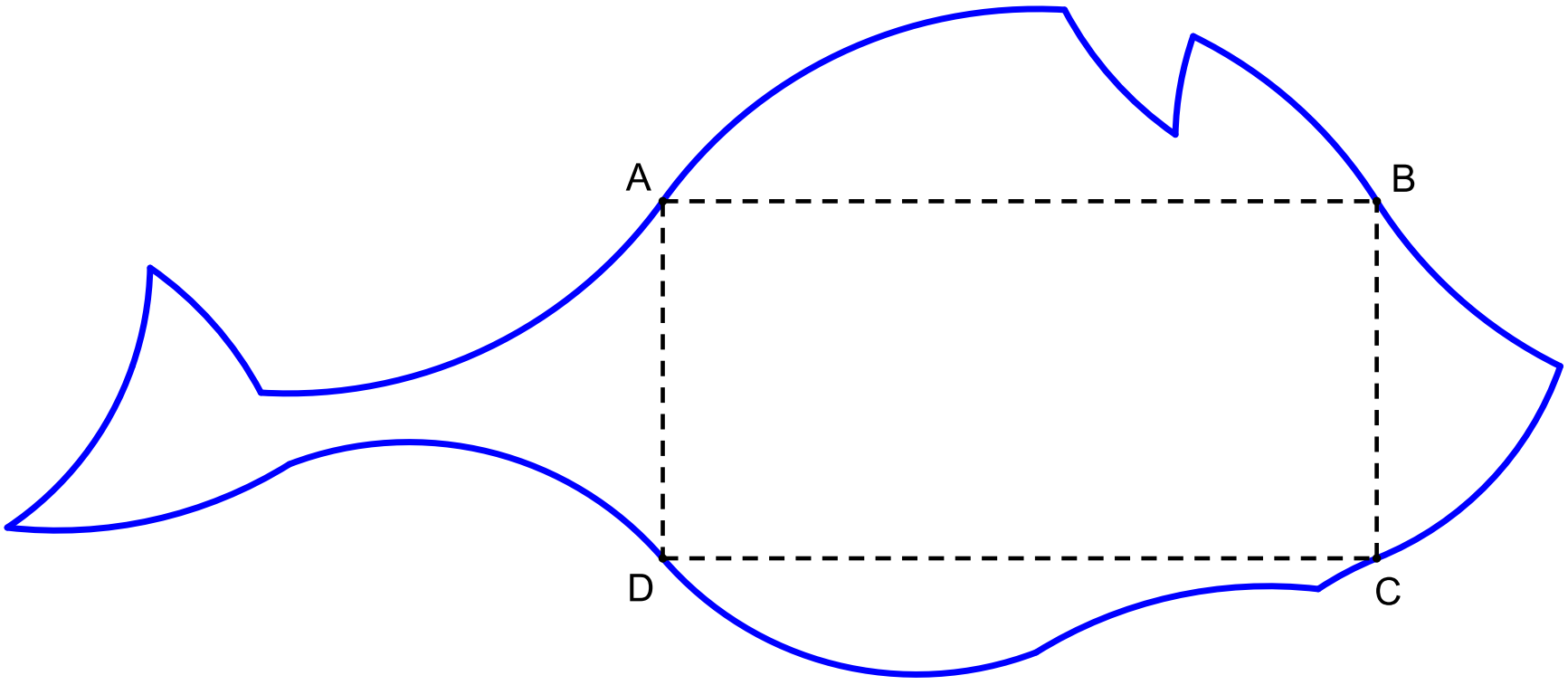


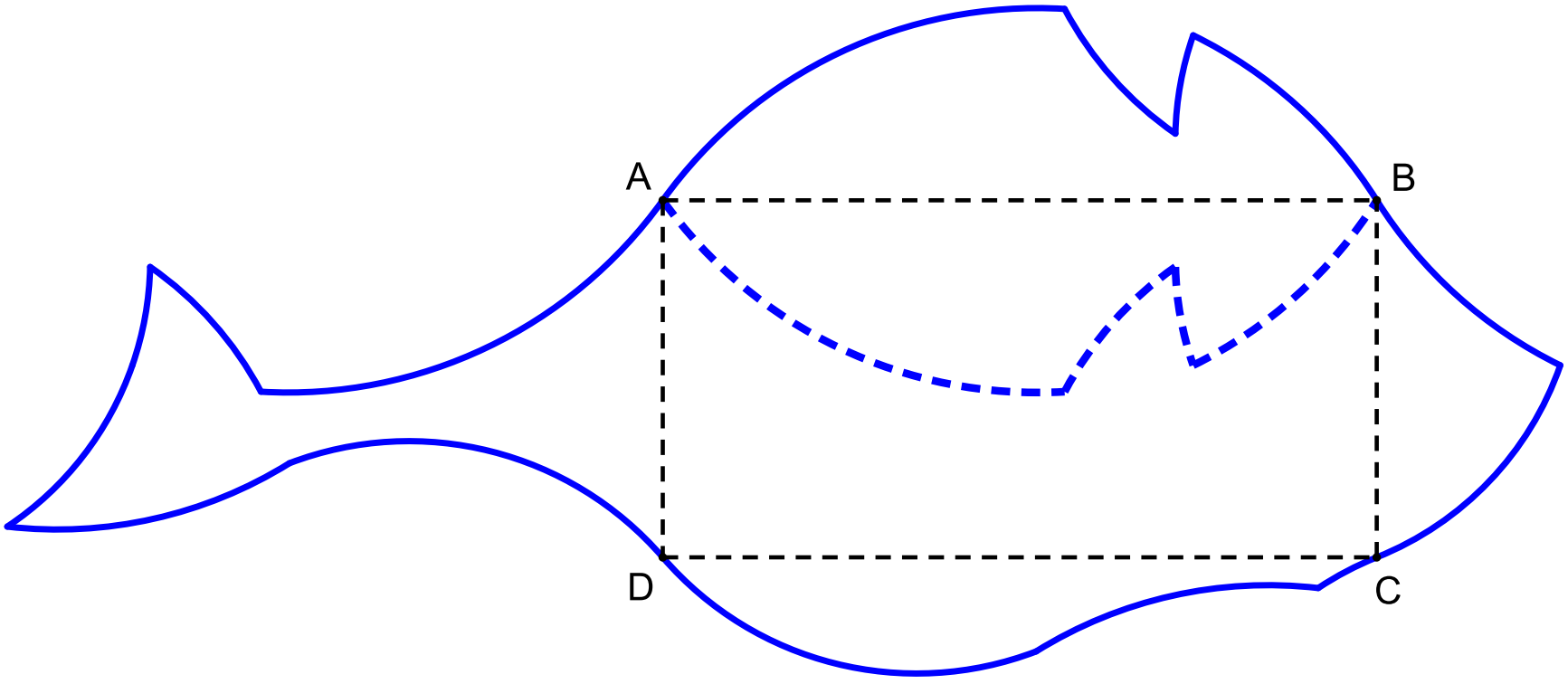


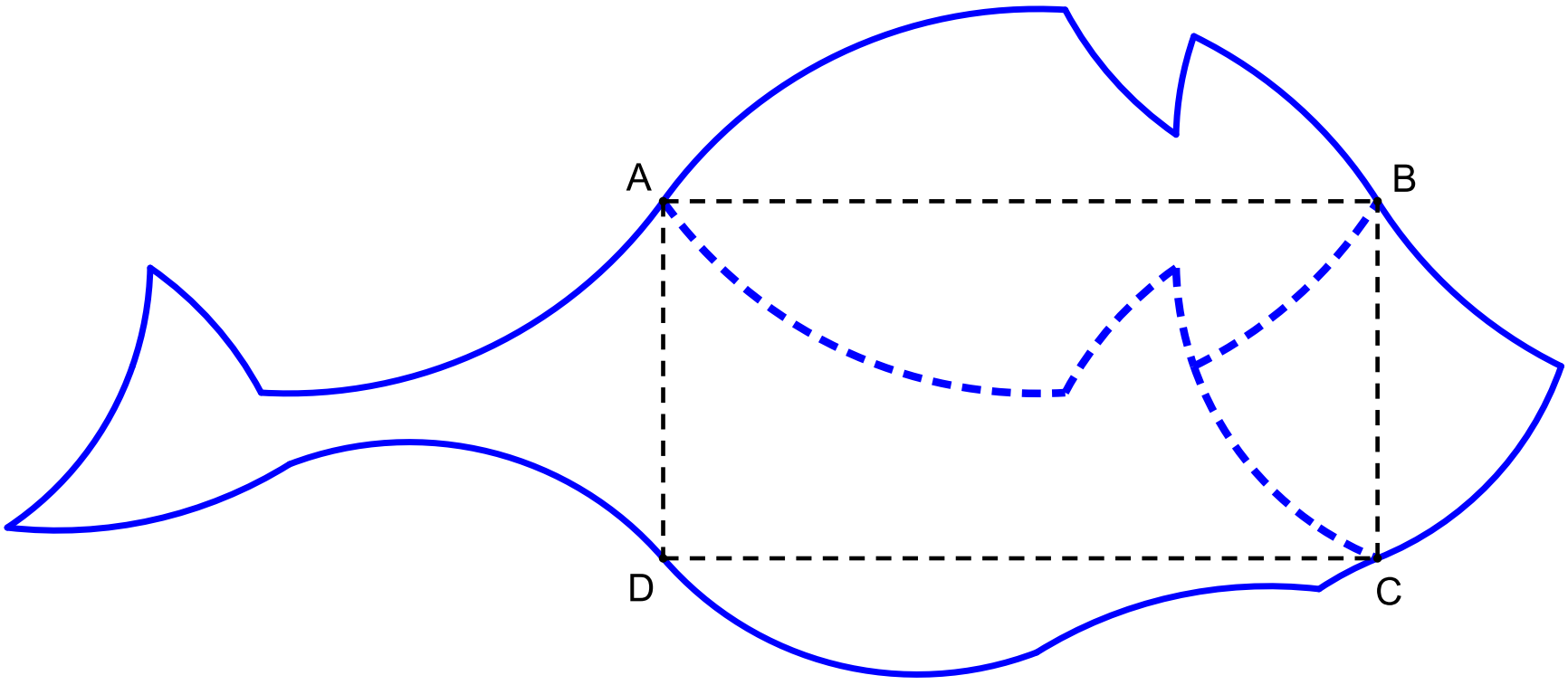
Heure symétrique !

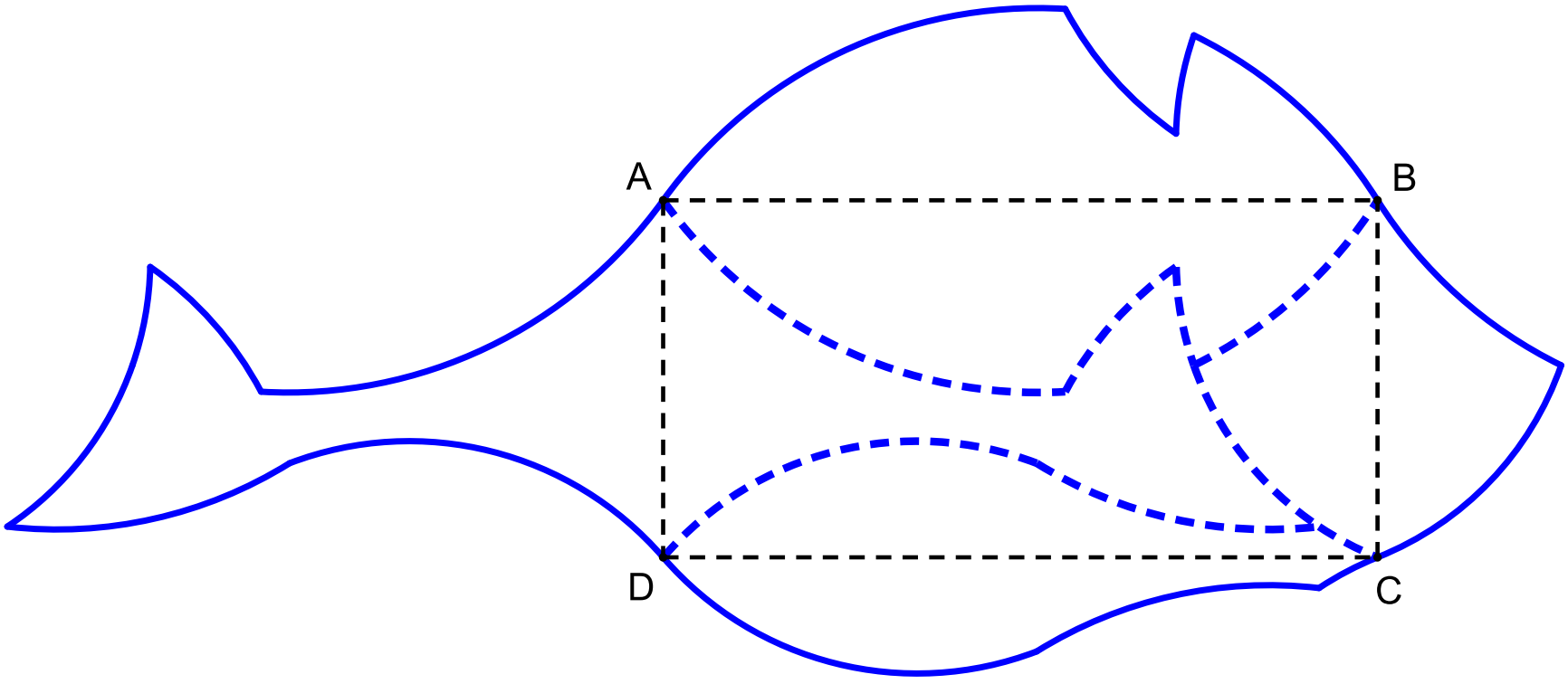
Mangez du poisson !



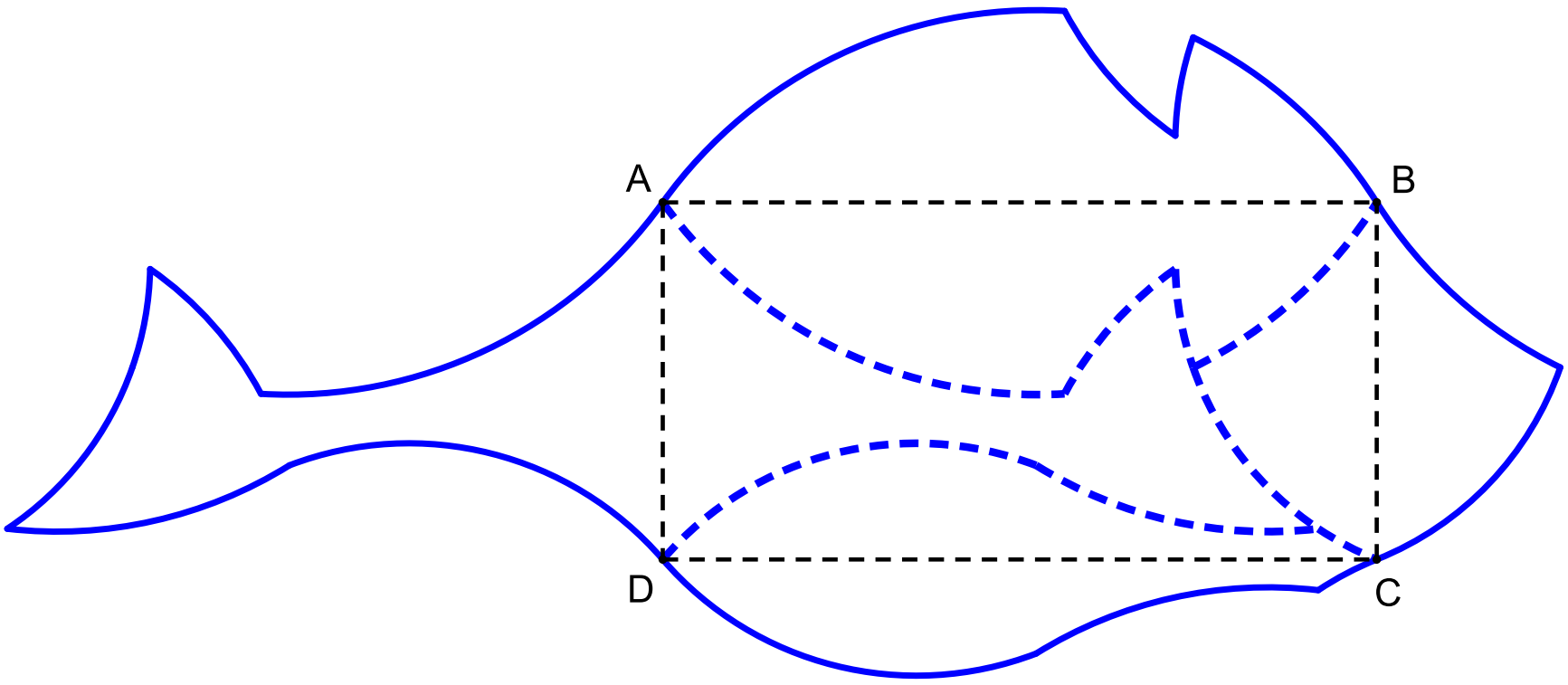

A
B
D
C

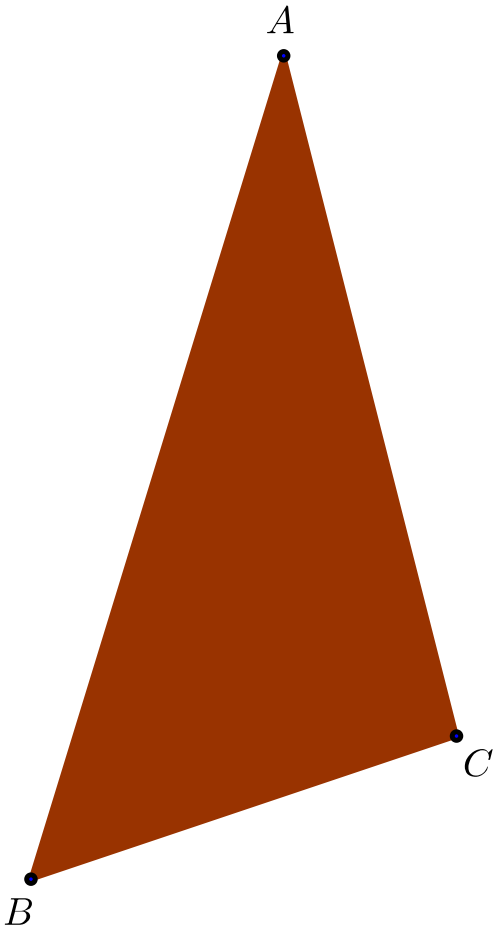

A
B
D
C

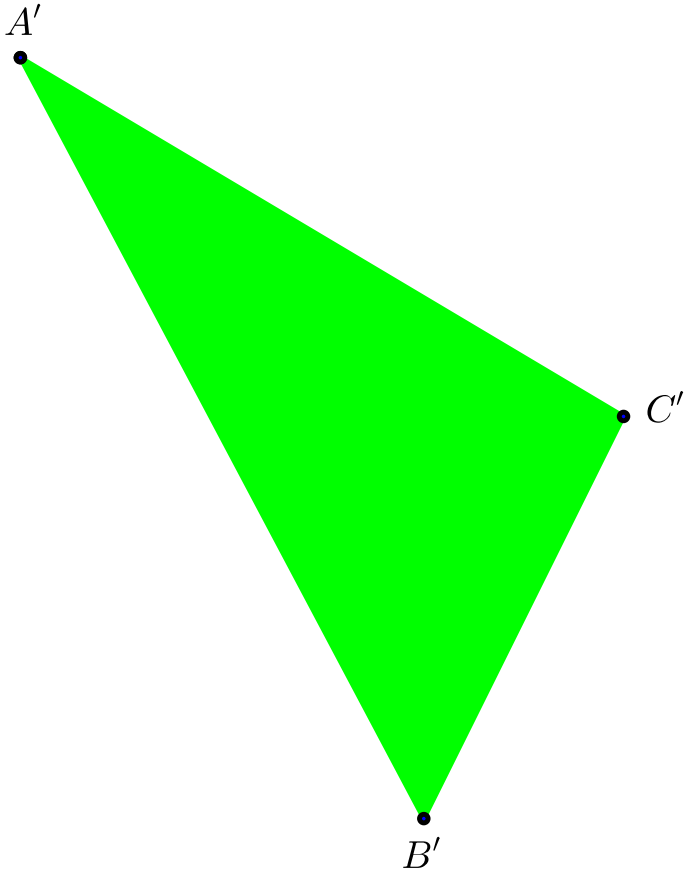

A
B
D
C


A
B
D
C

Le rectangle $ABCD$ a pour longueur 20cm et pour largeur 10cm.
Quelle est l'aire du poisson ?



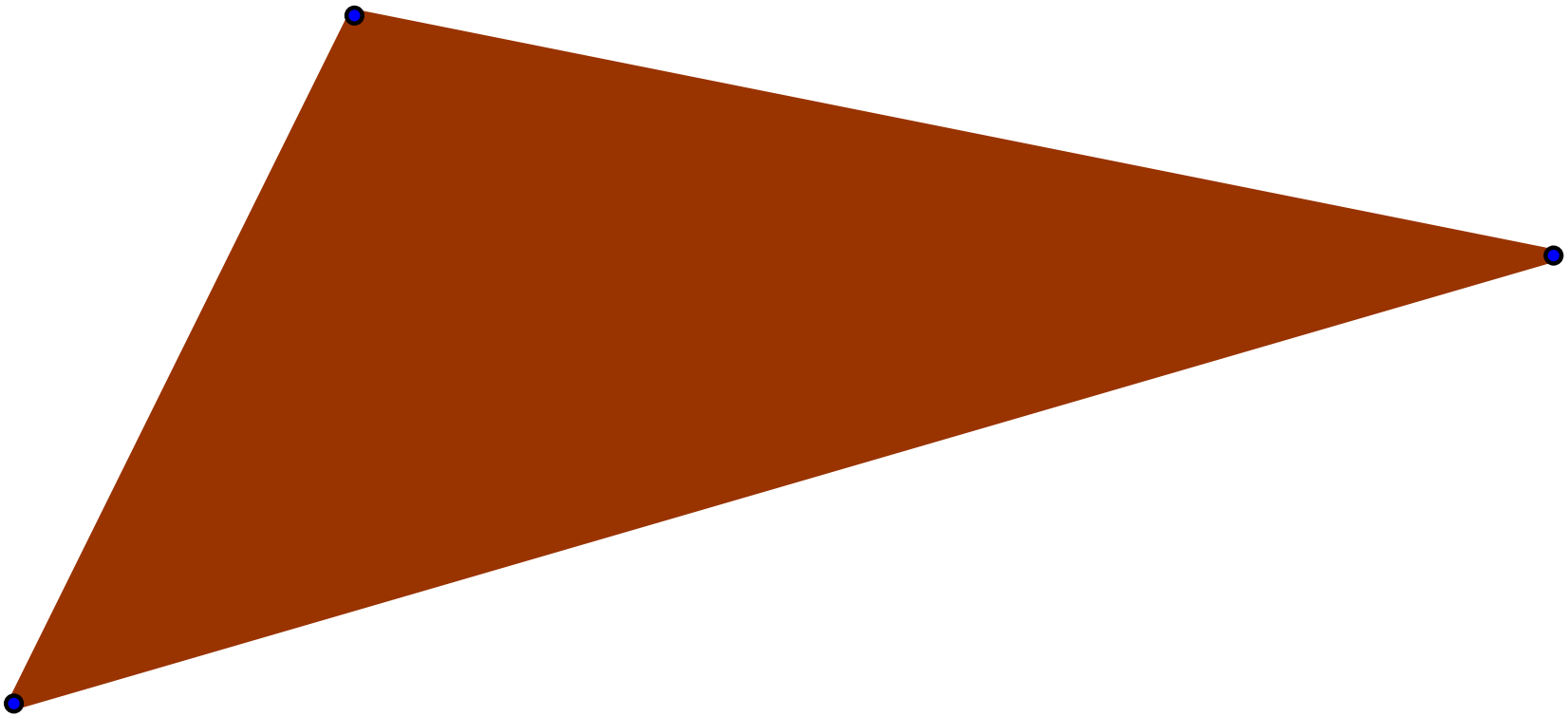

A
C
B

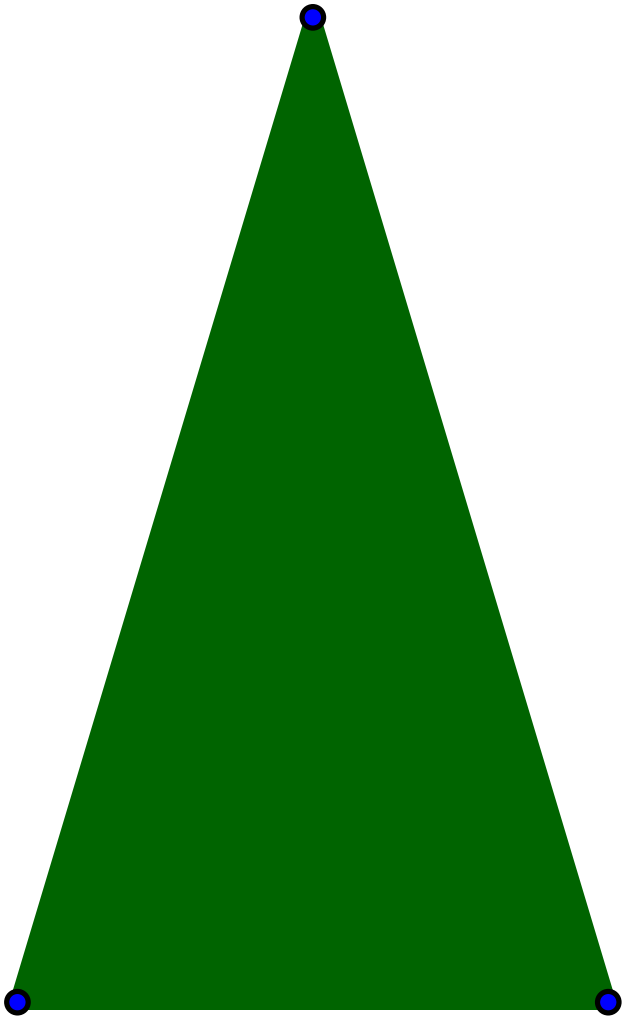

A′
C′
B′


TrocVestiaire.com
37 ideal
37 ideal

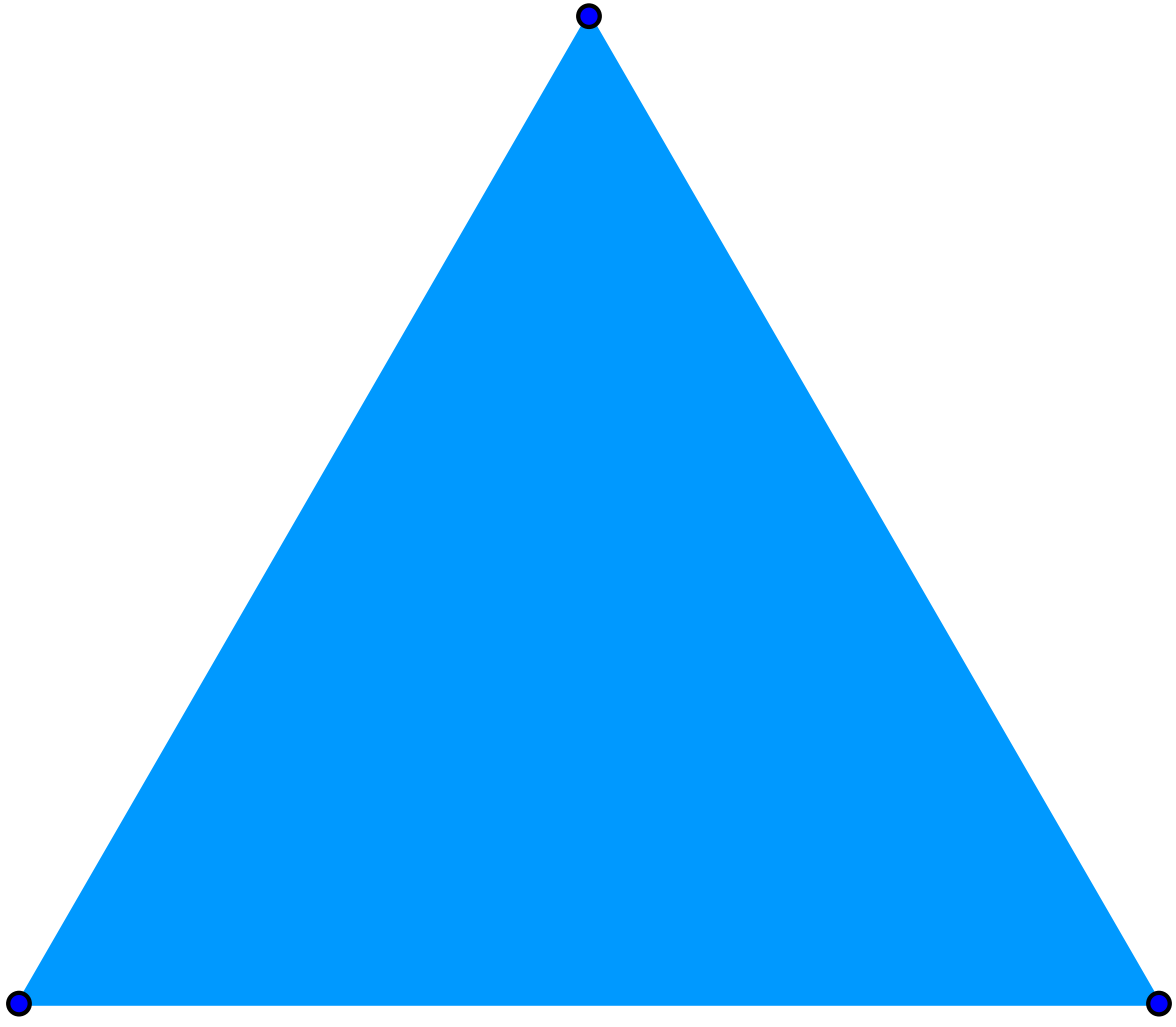
3. Figures géométriques

Des triangles

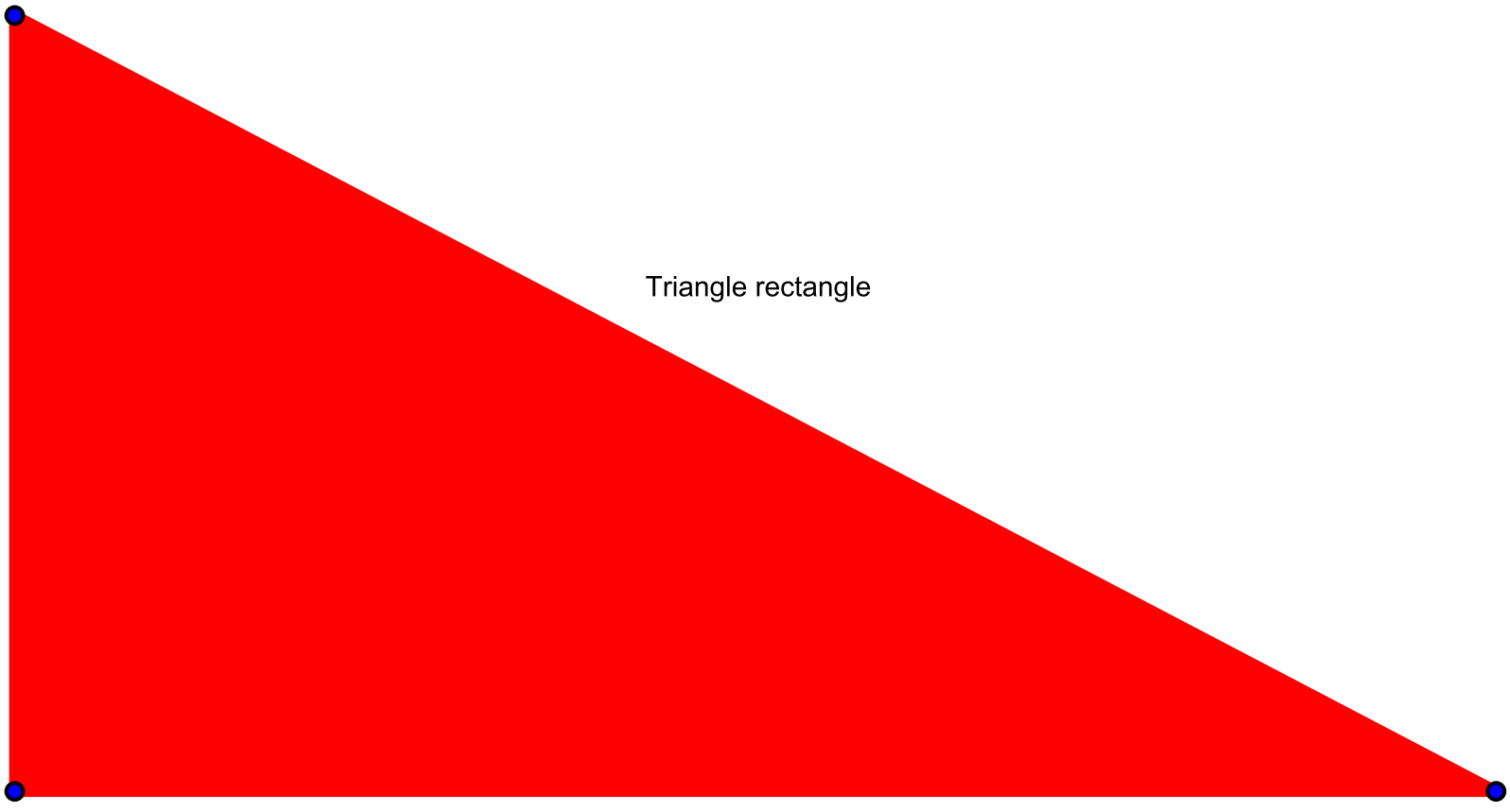


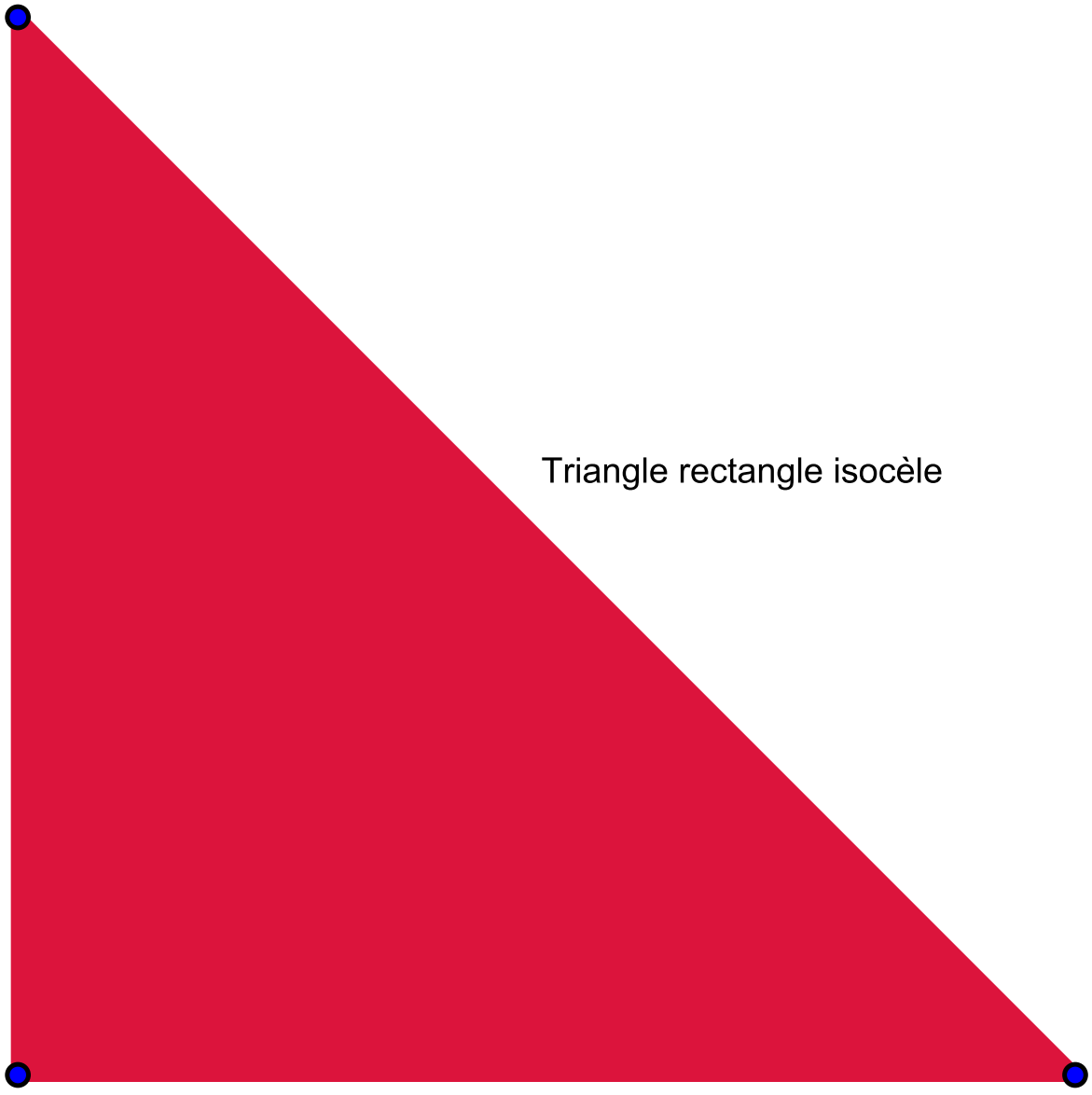


Triangle isocèle



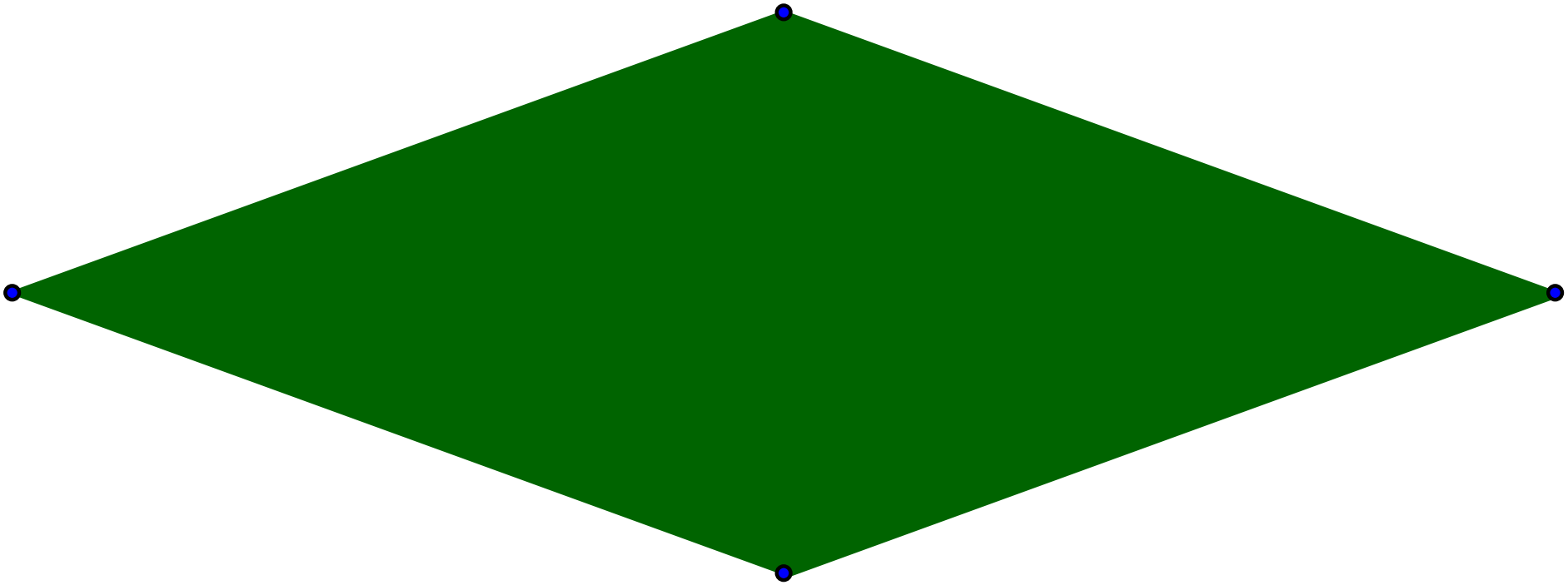
Triangle équilatéral


Triangle rectangle

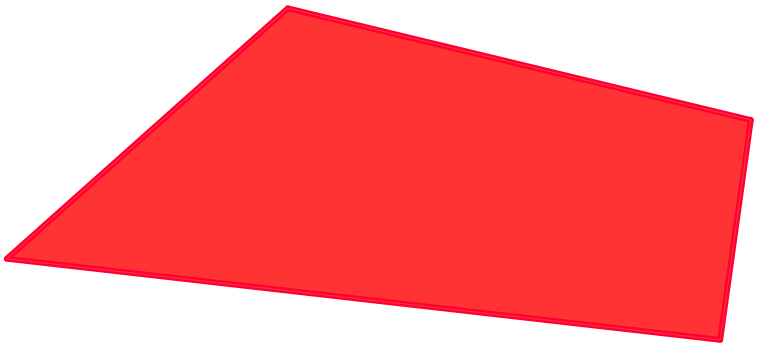

Triangle rectangle isocèle

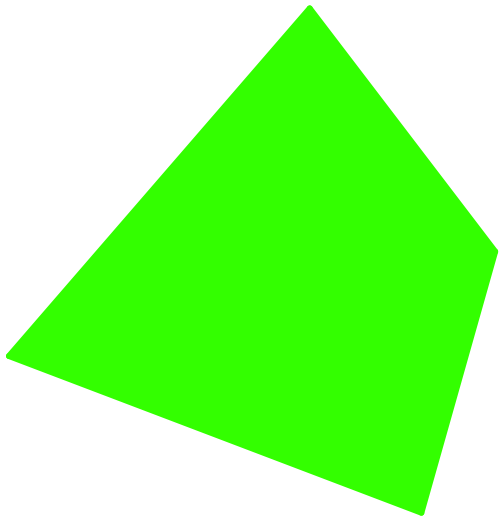
Des quadrilatères

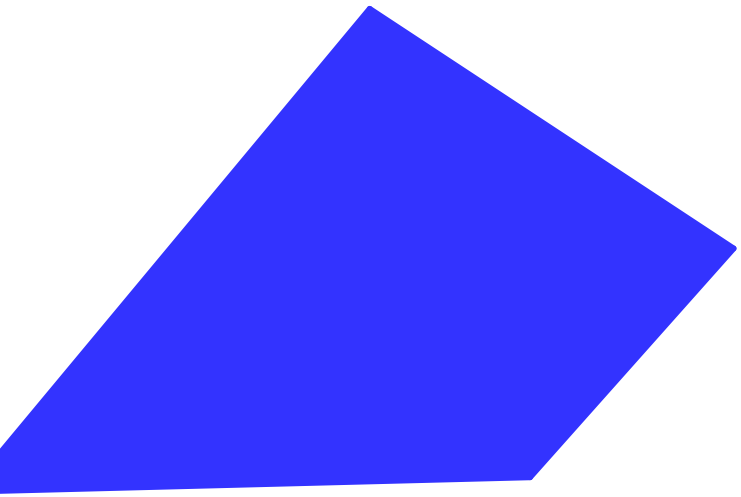
Parallélogramme

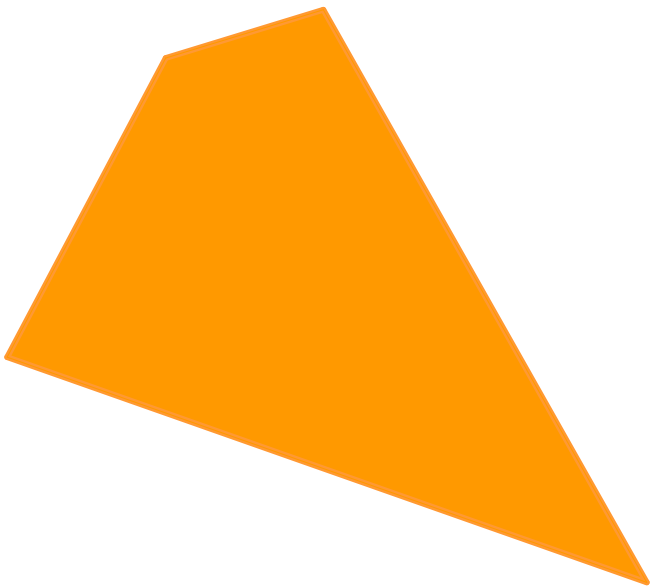


Losange

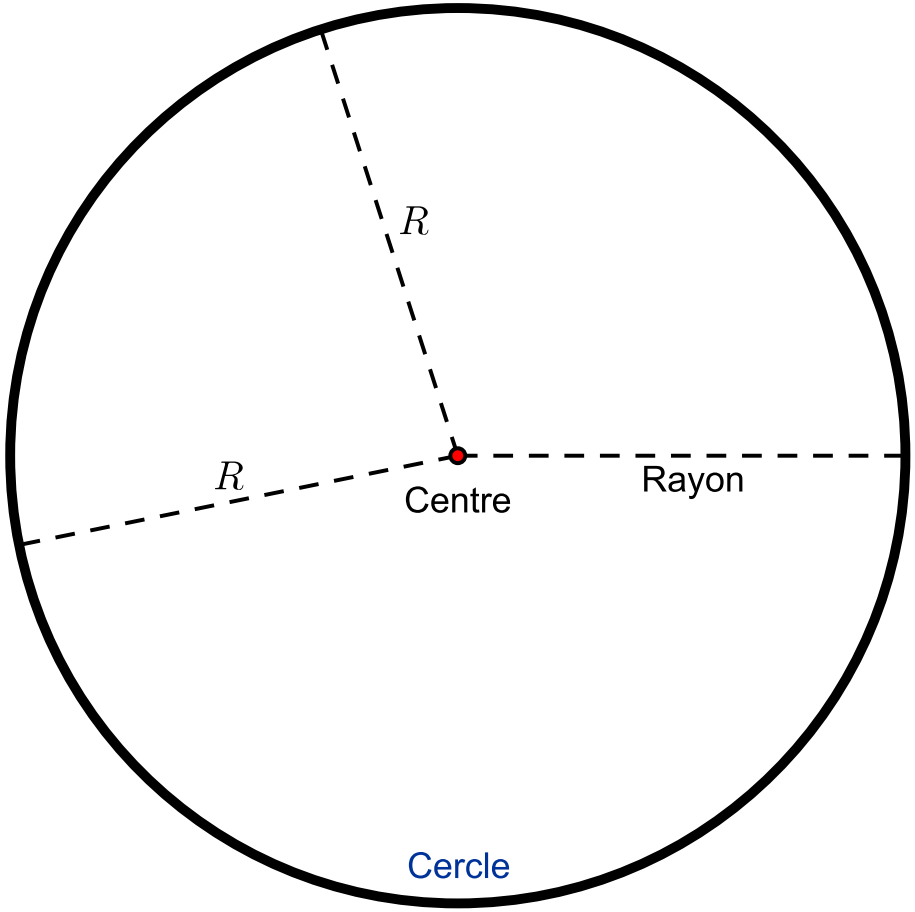




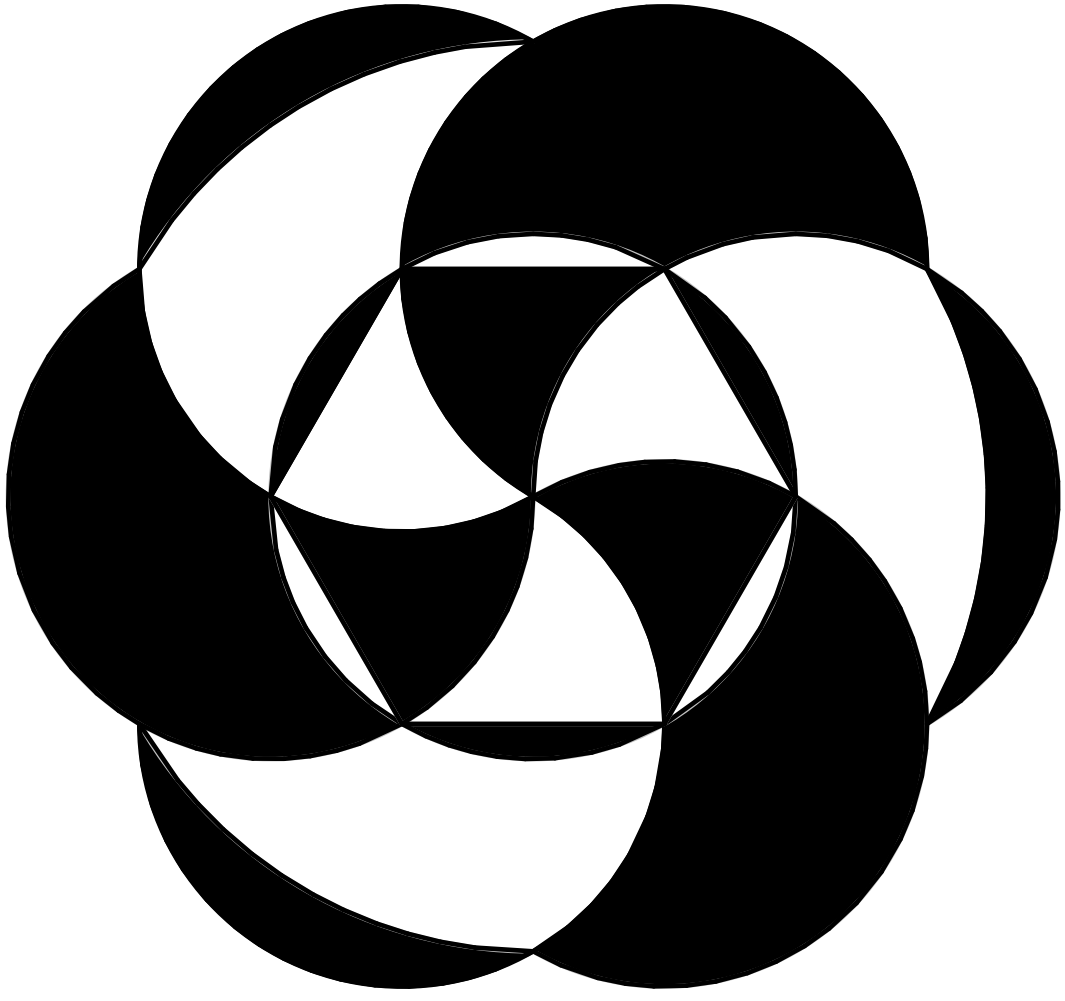




Le cercle



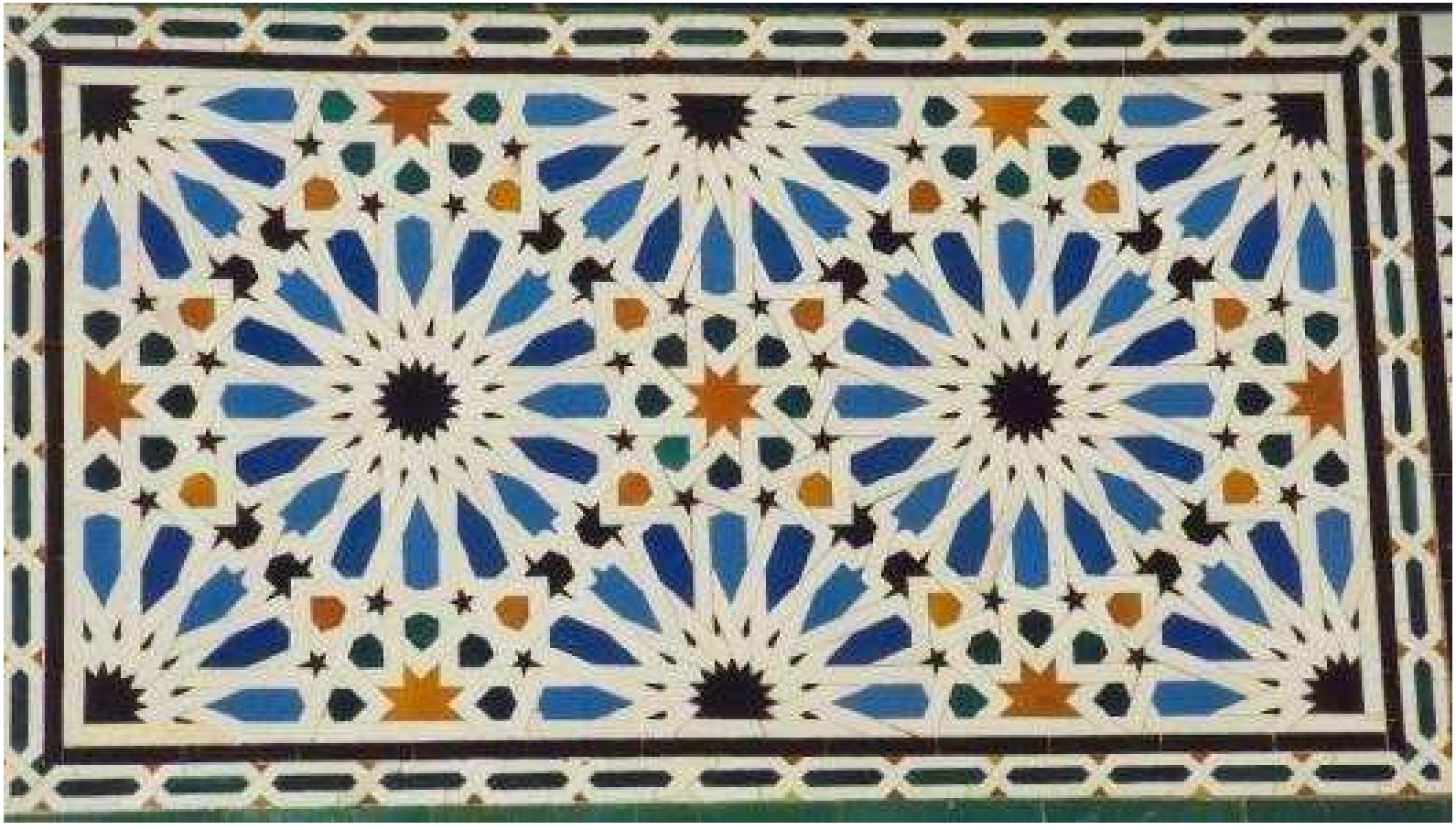
Le cercle Γ de centre O et de rayon $R \geq 0$ est l'ensemble des points dont la distance à O vaut R !

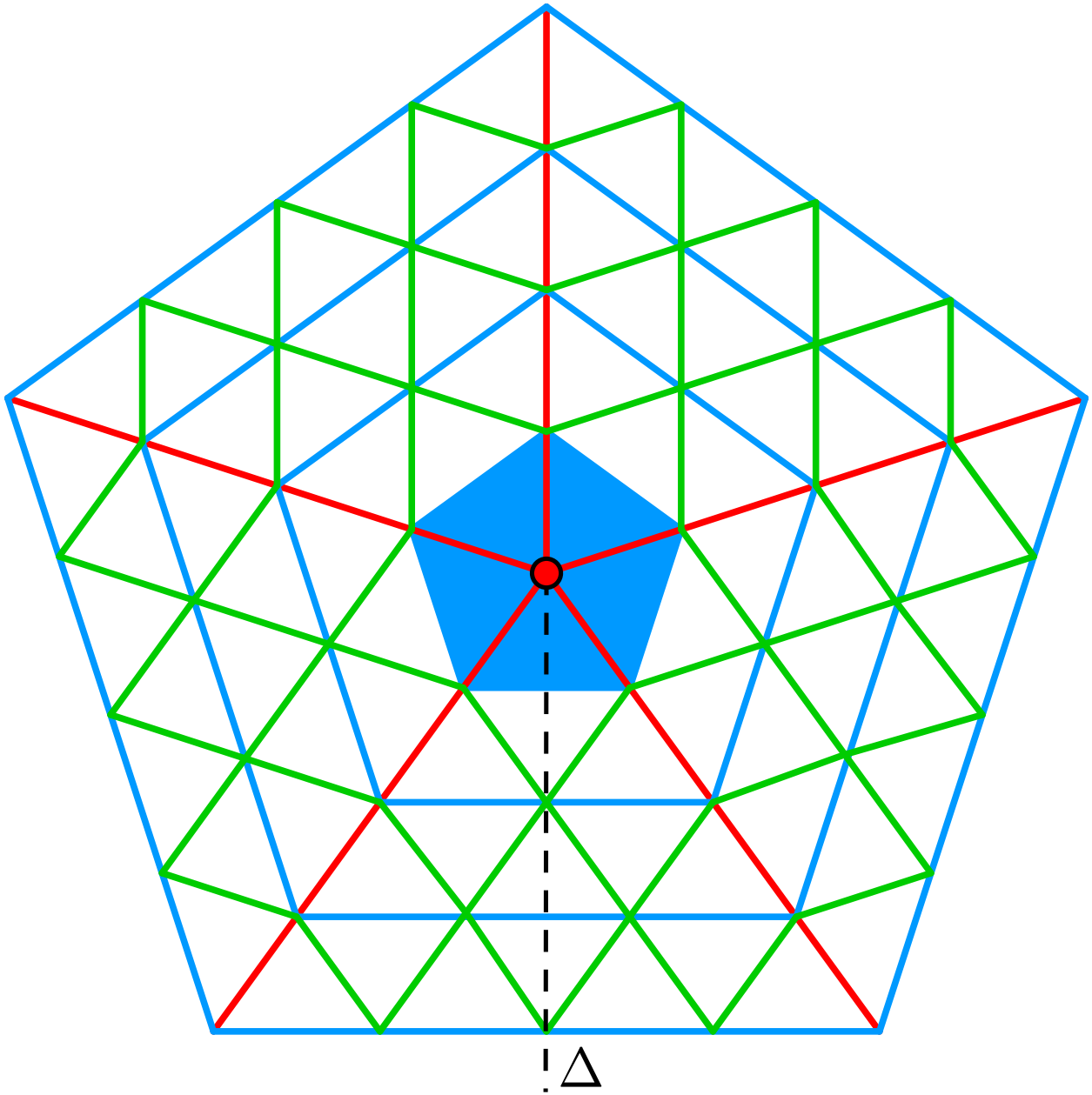


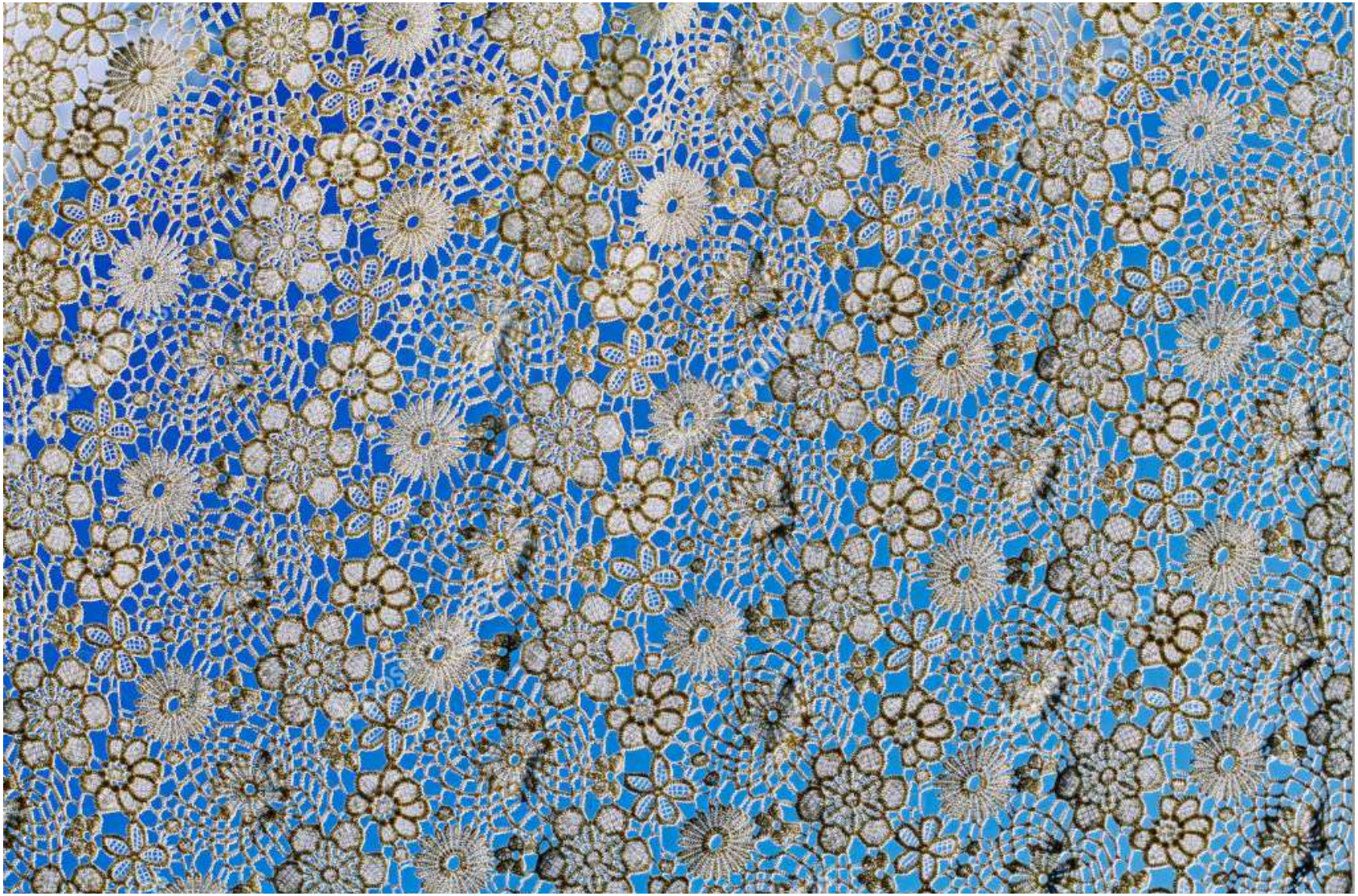
On peut fabriquer de belles figures avec des arcs de cercle !

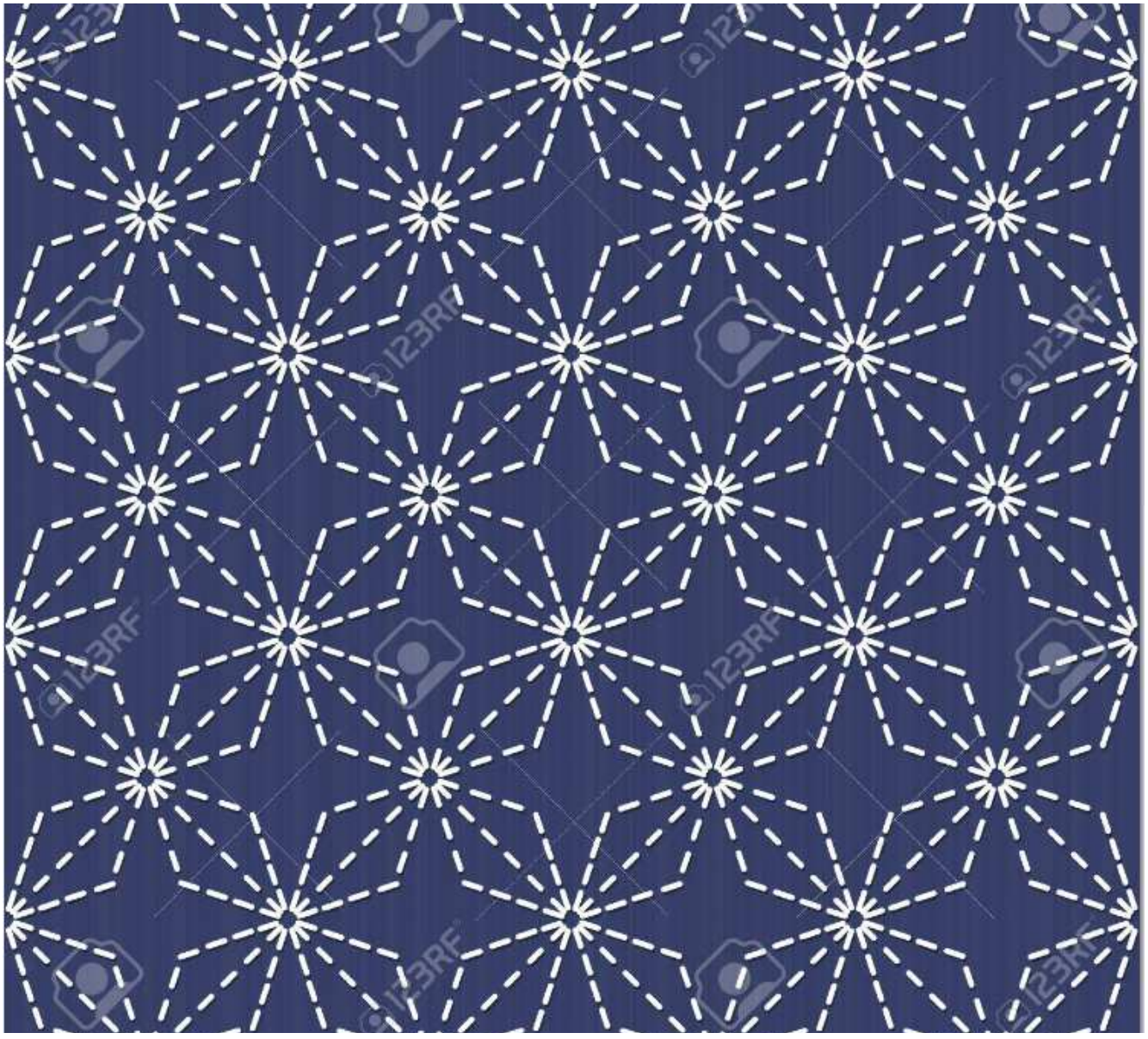
4. Pavages

Quelques exemples

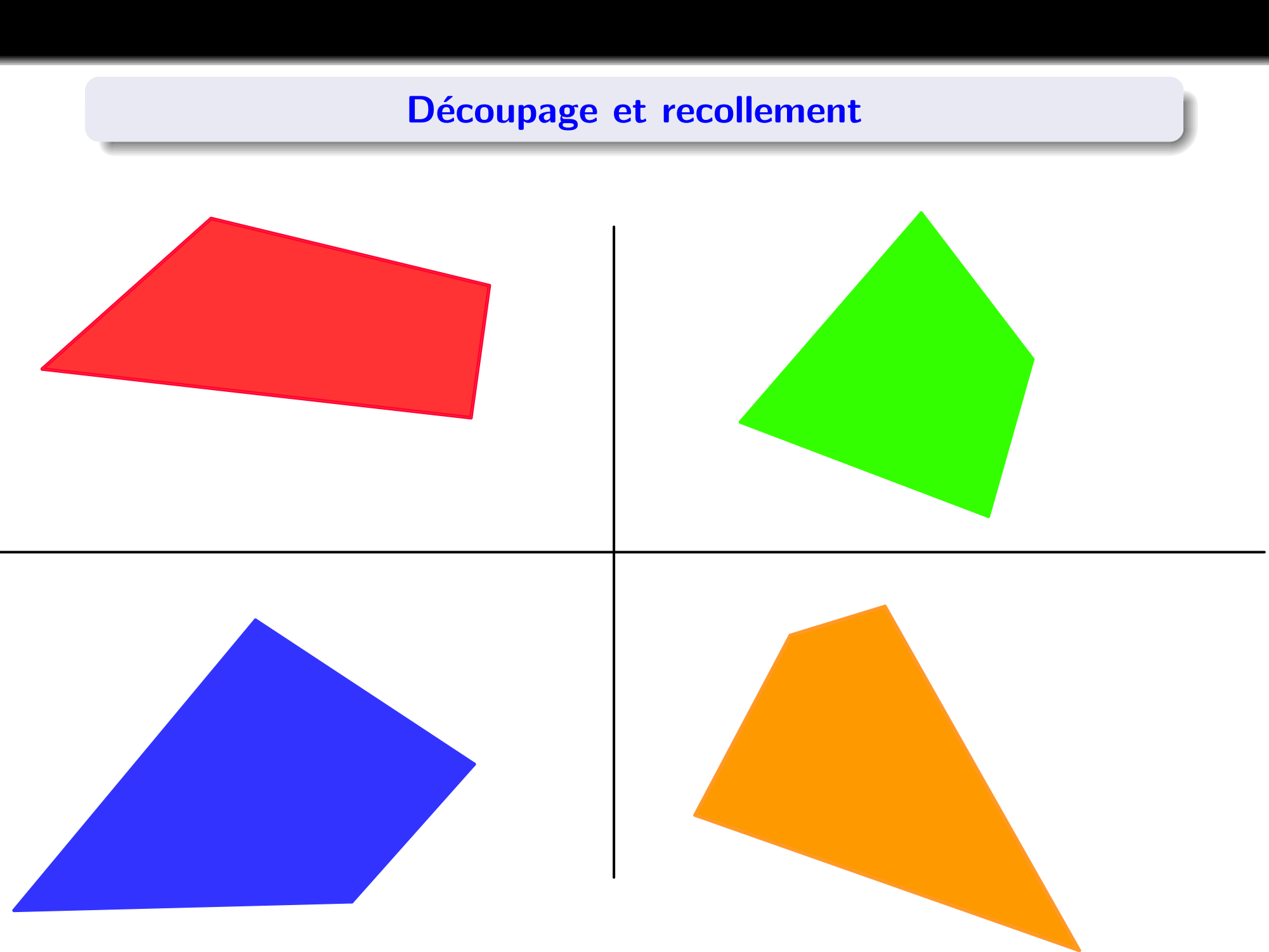


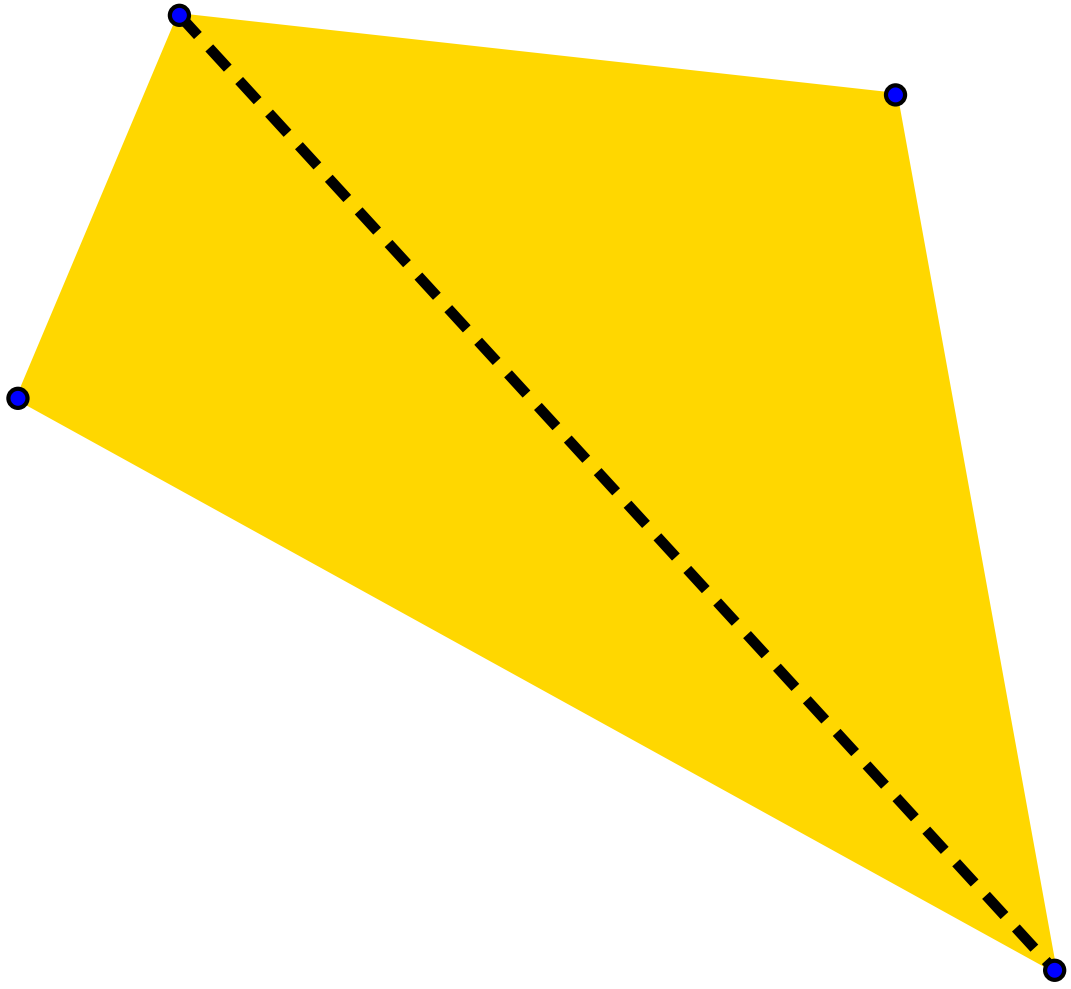

Δ



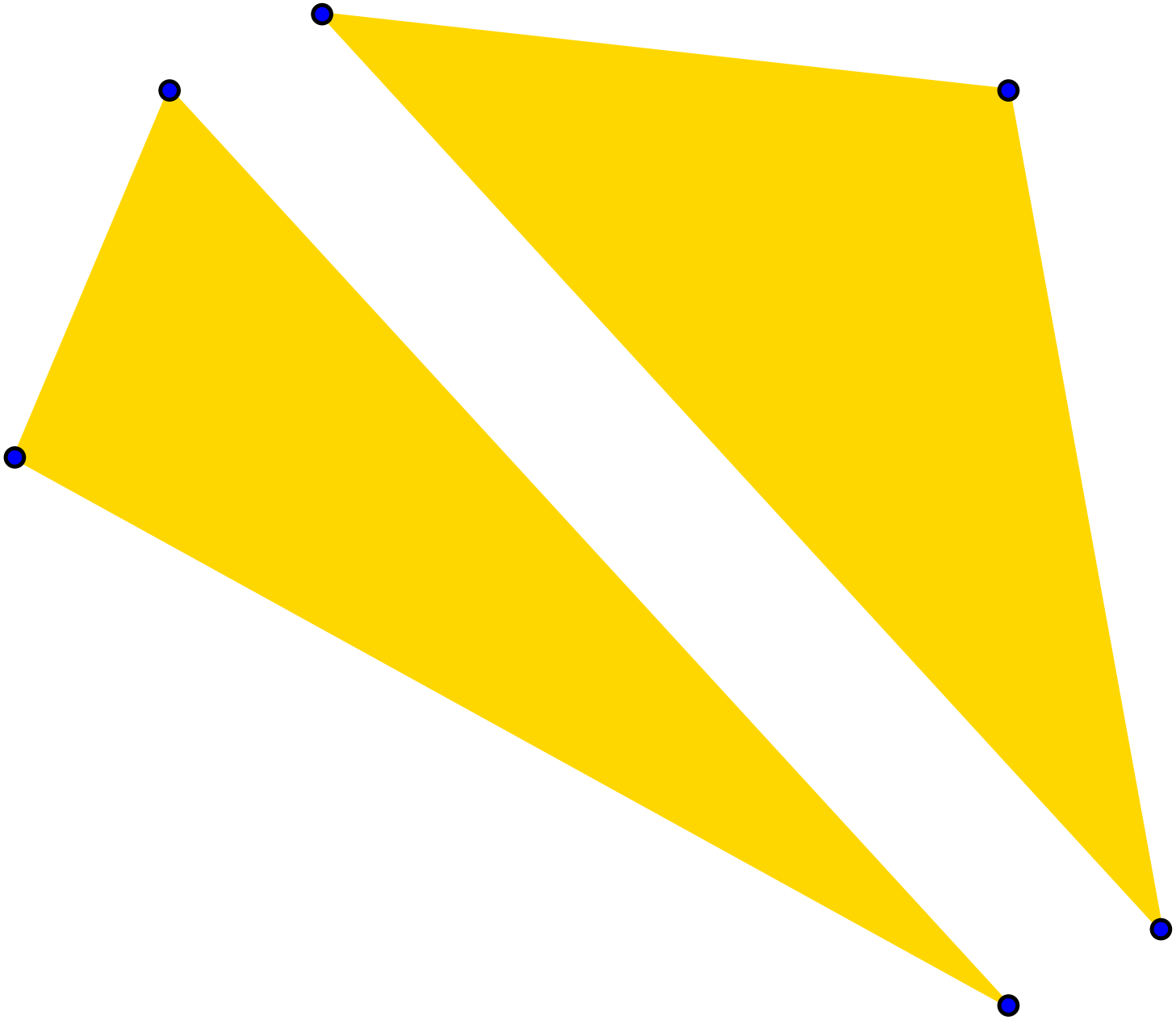


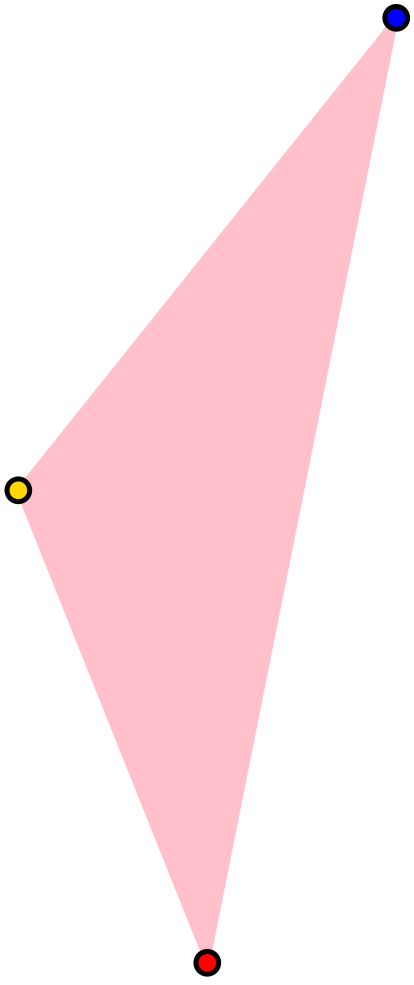
Découpage et recollement

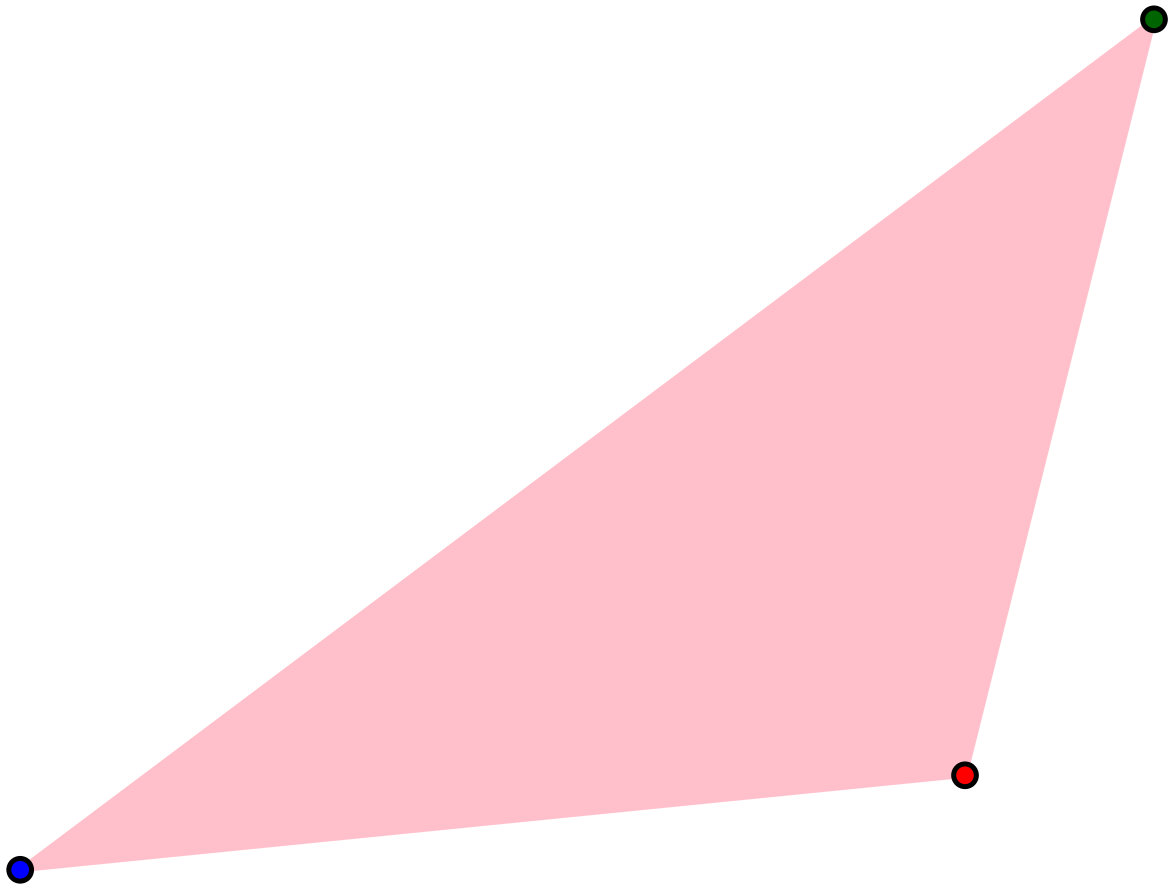


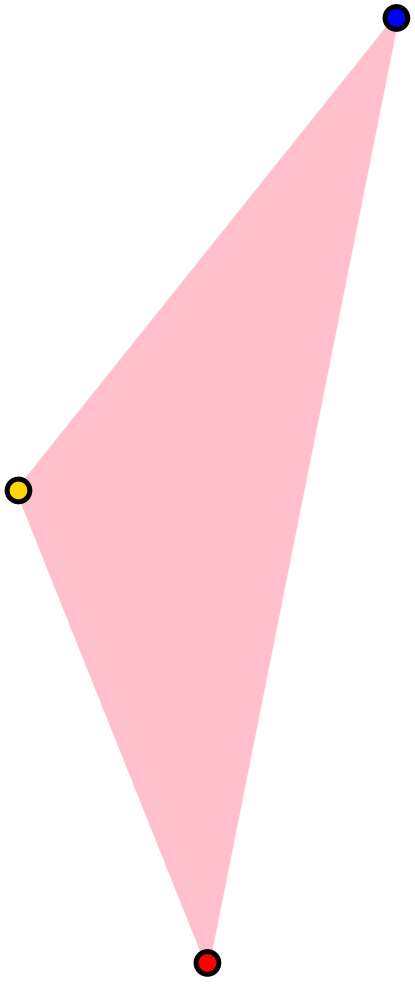


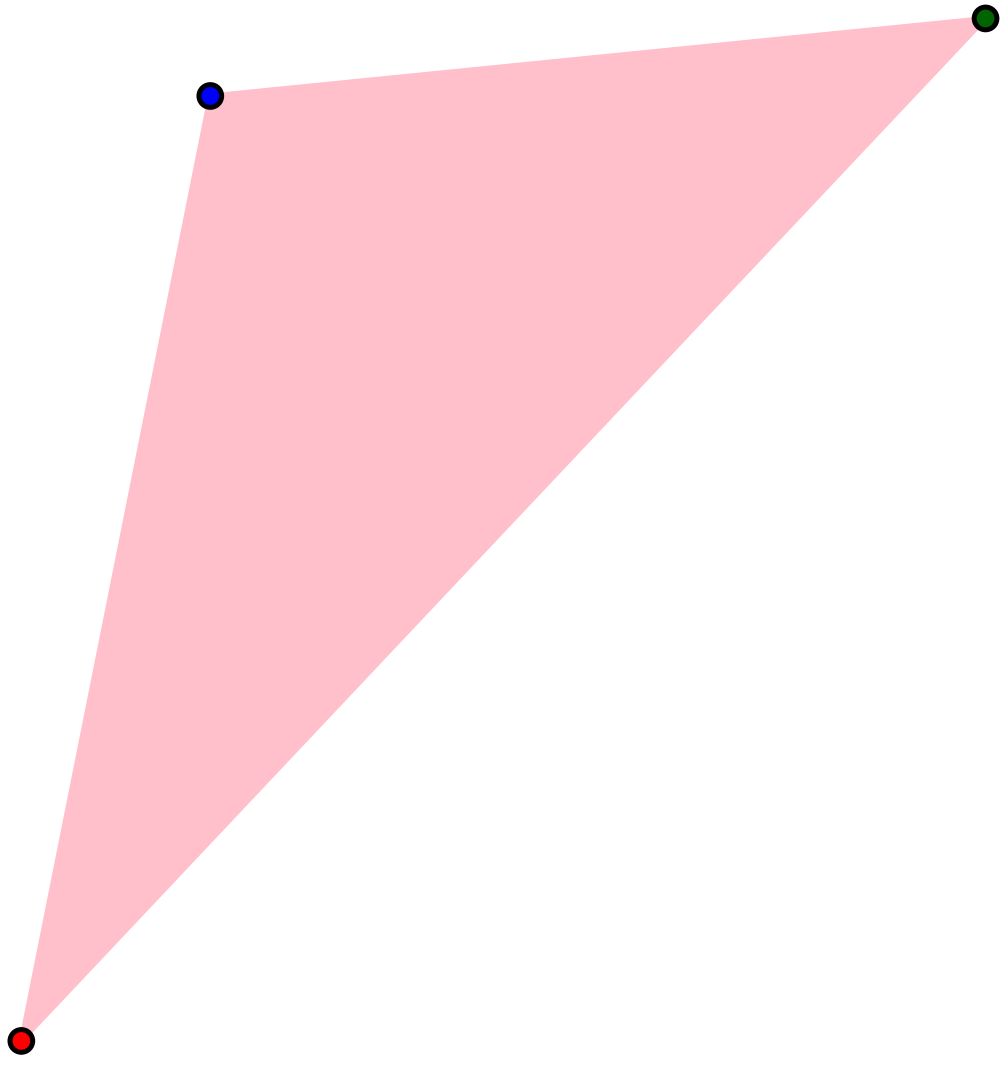
Quadrilatère quelconque et une diagonale

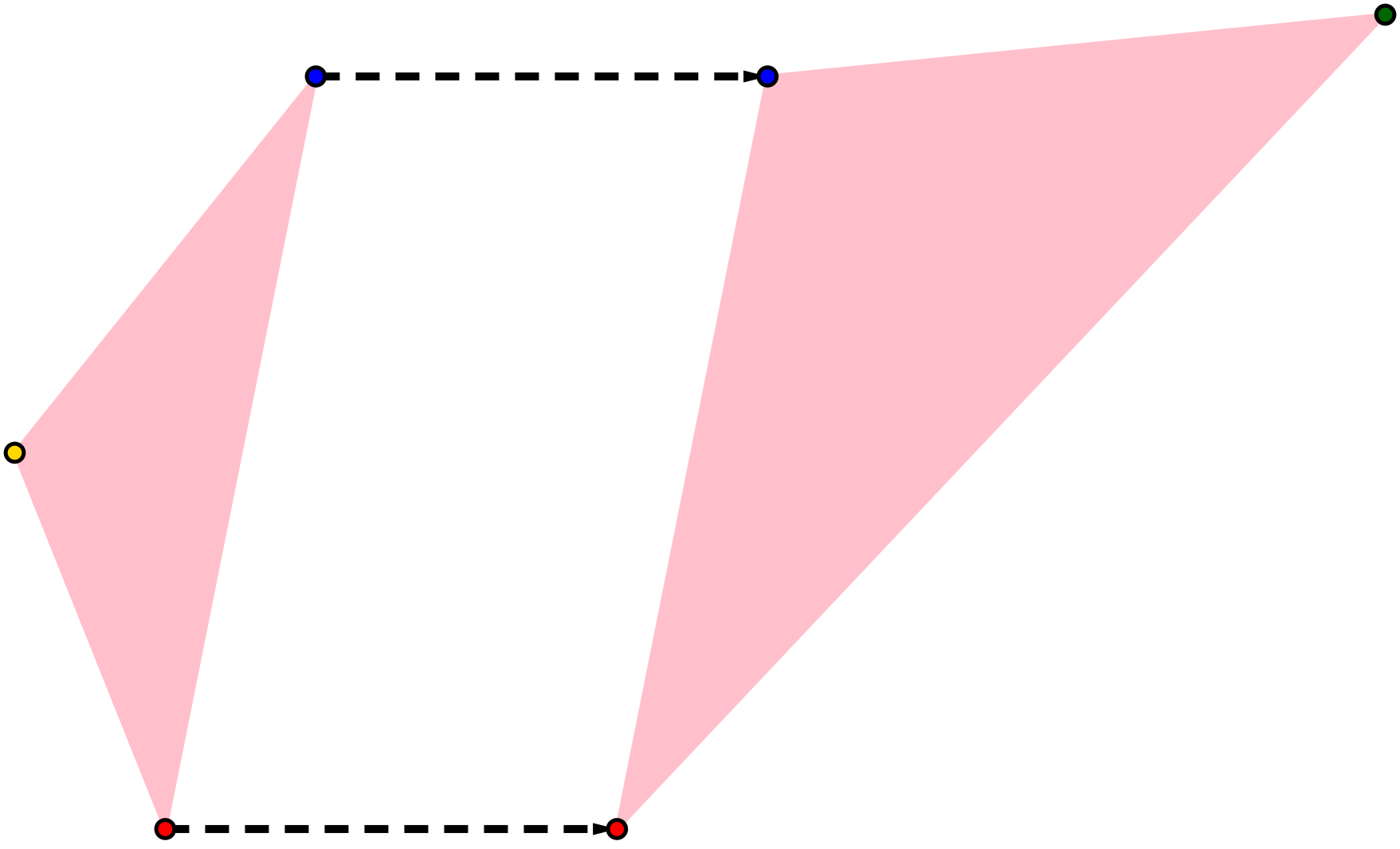


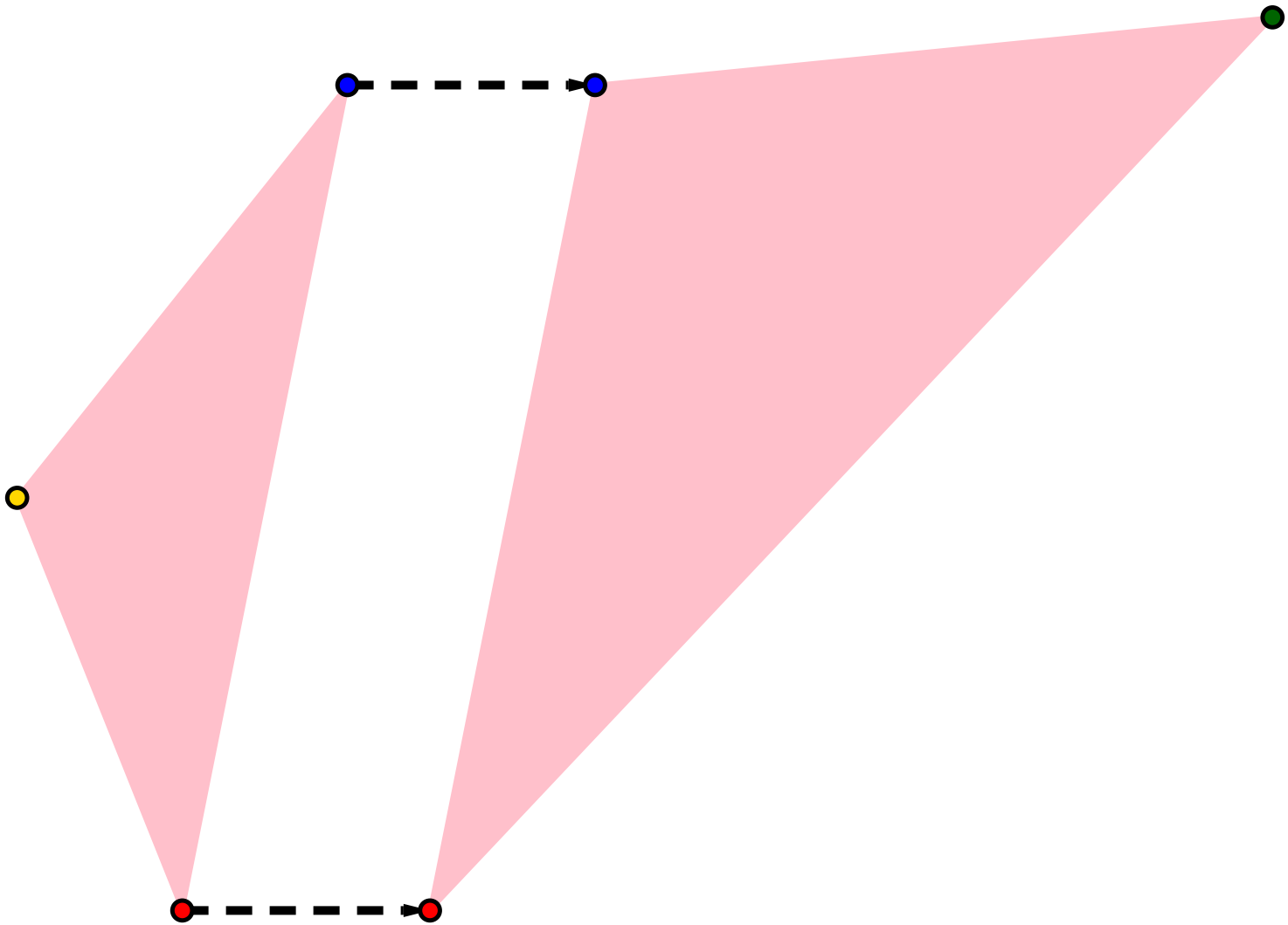


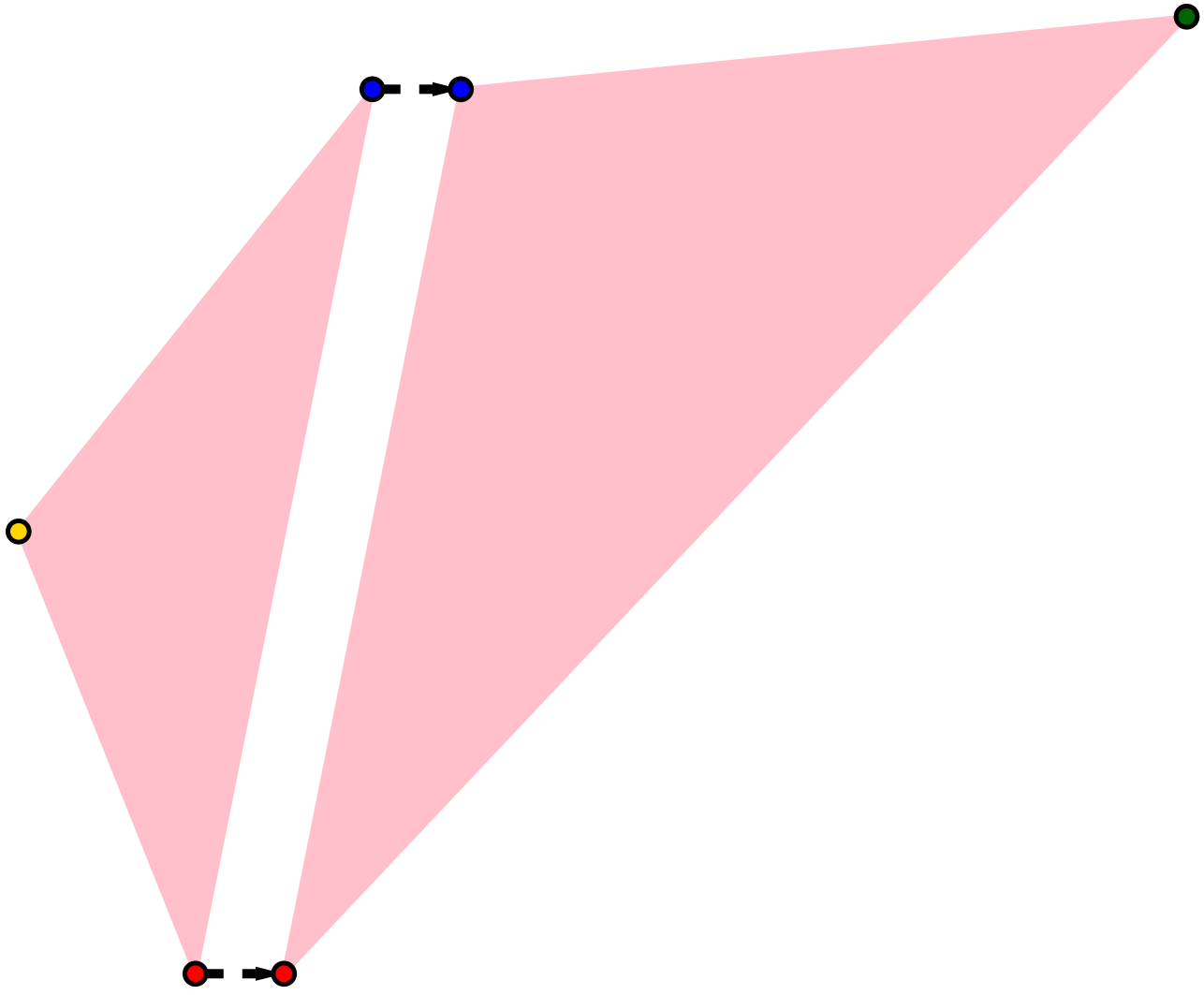


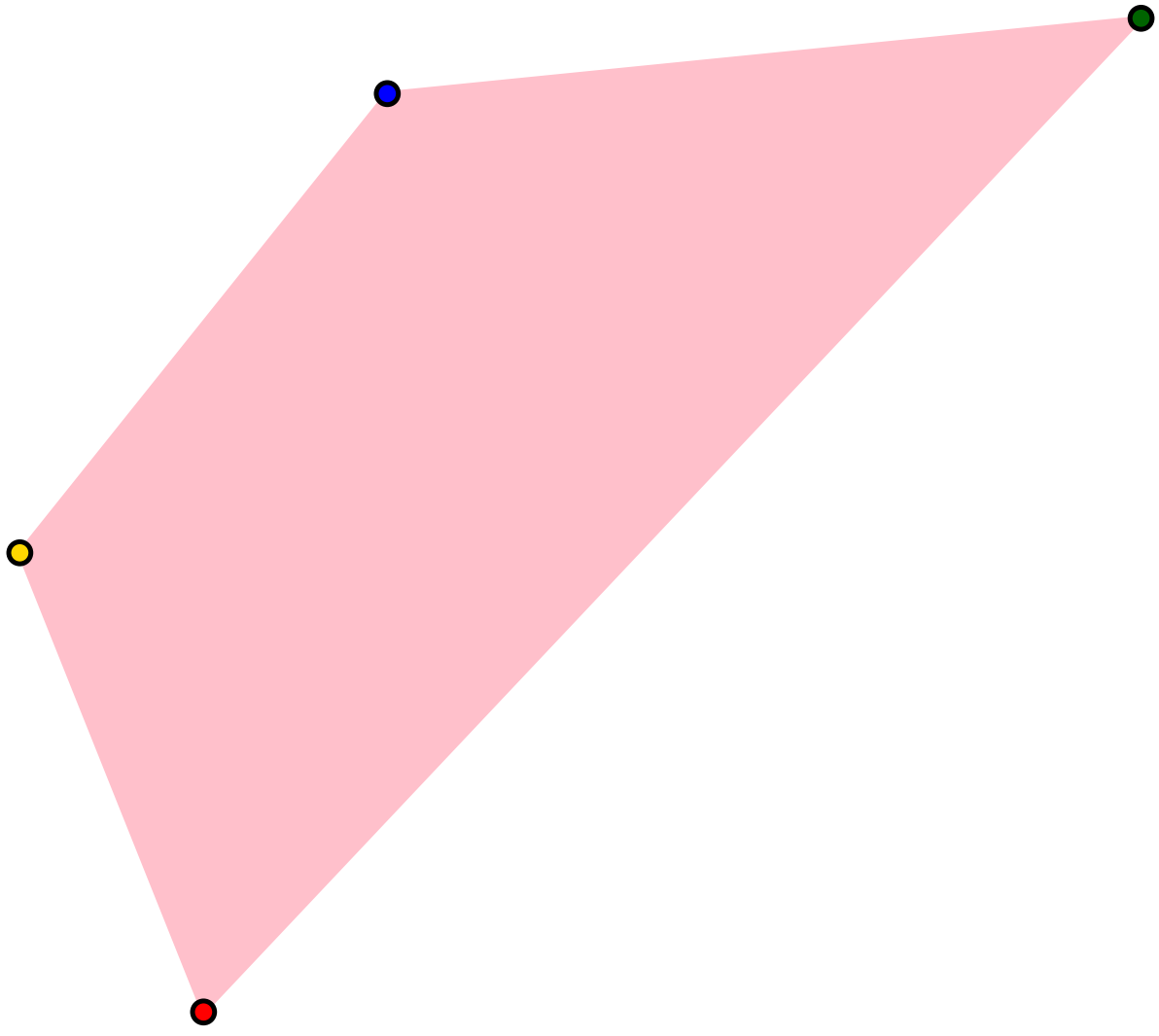


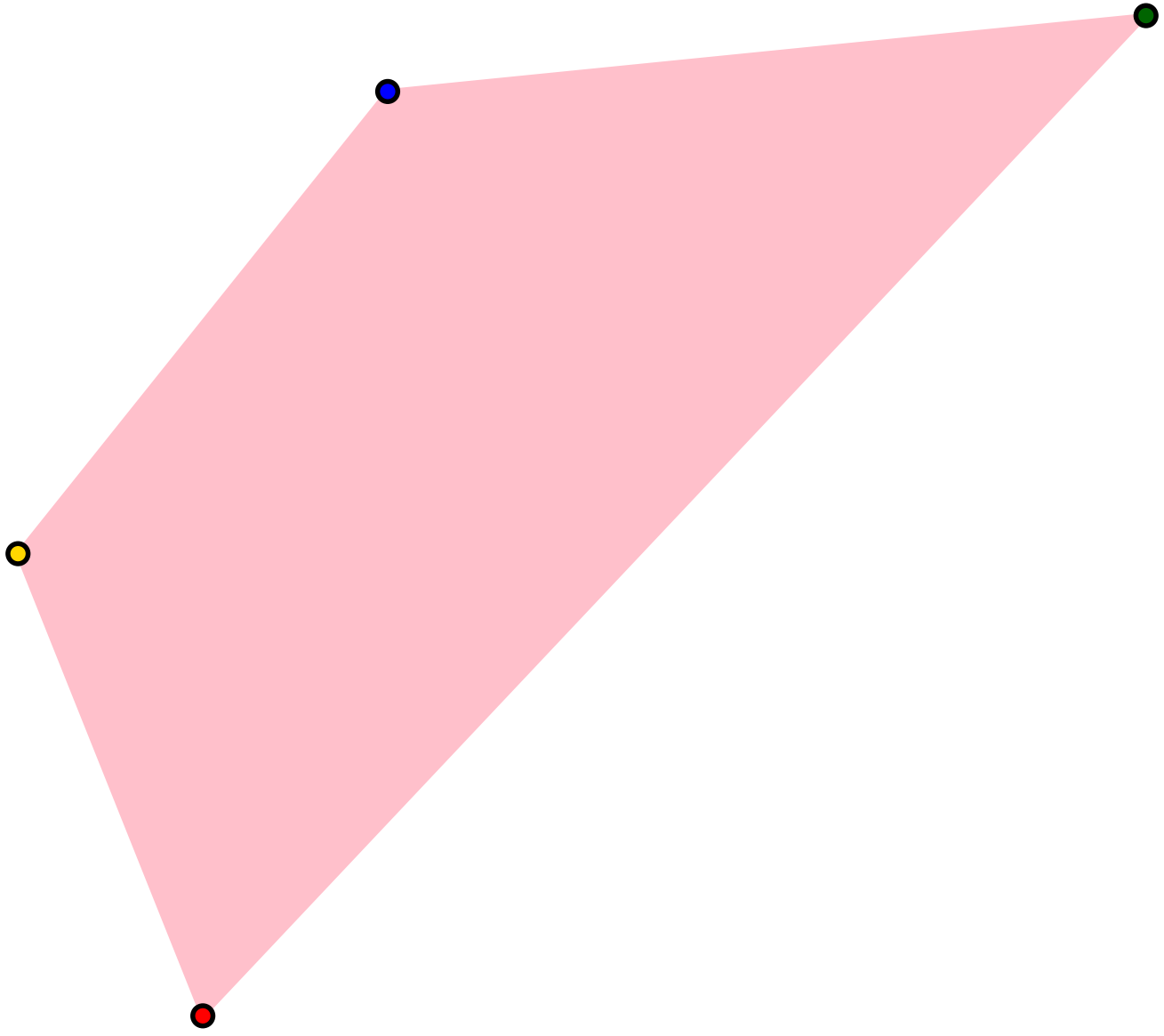


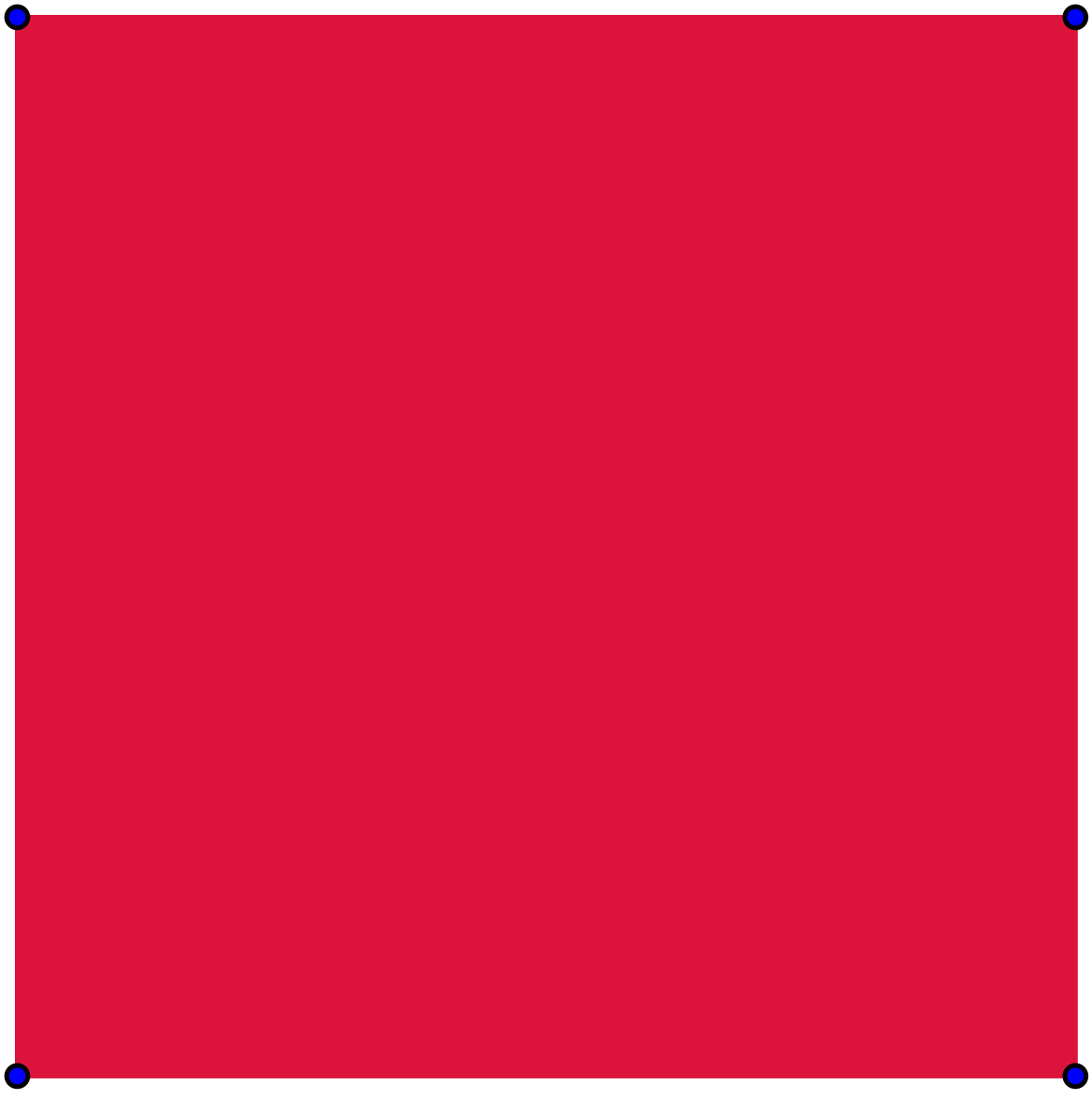


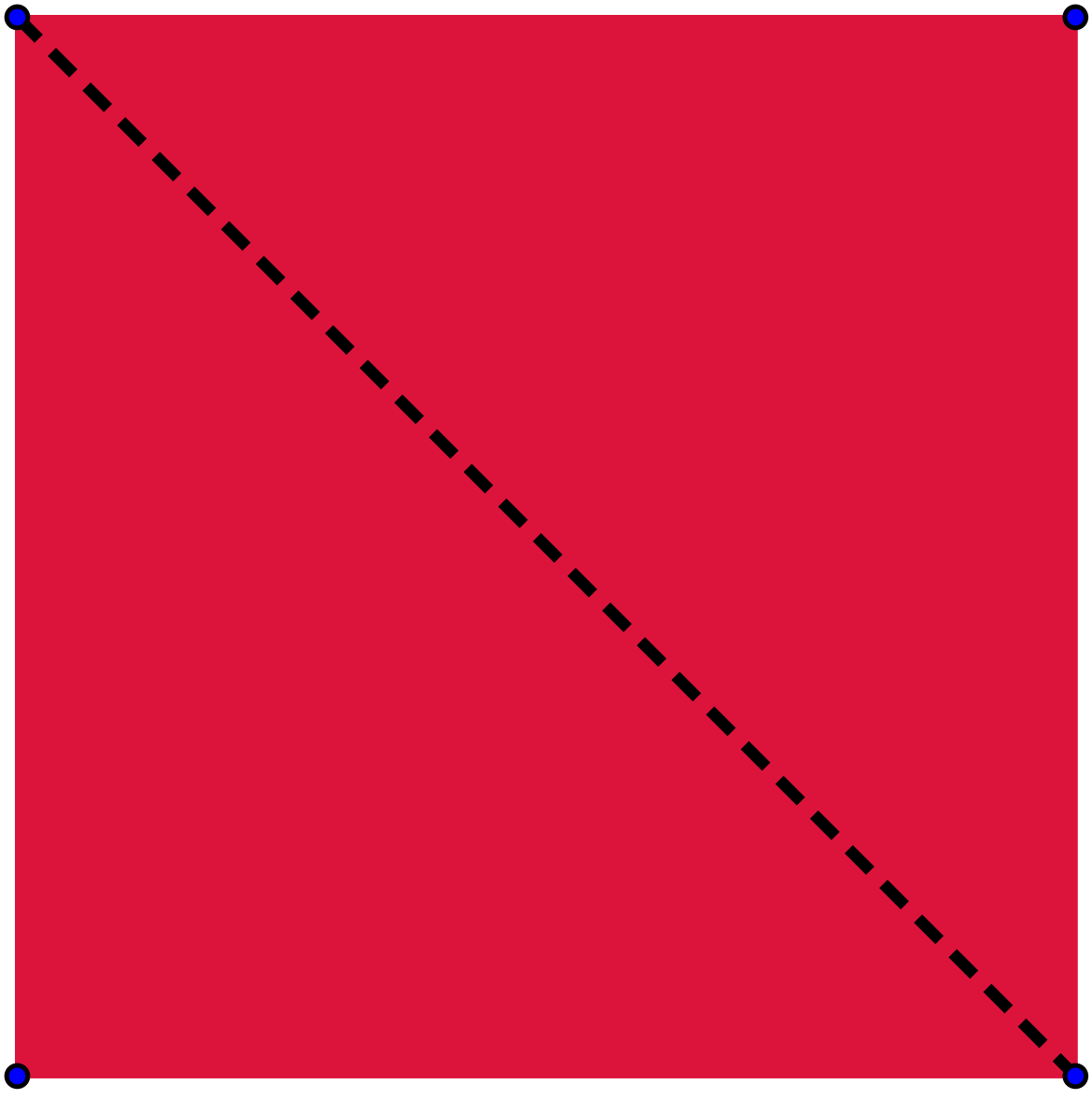


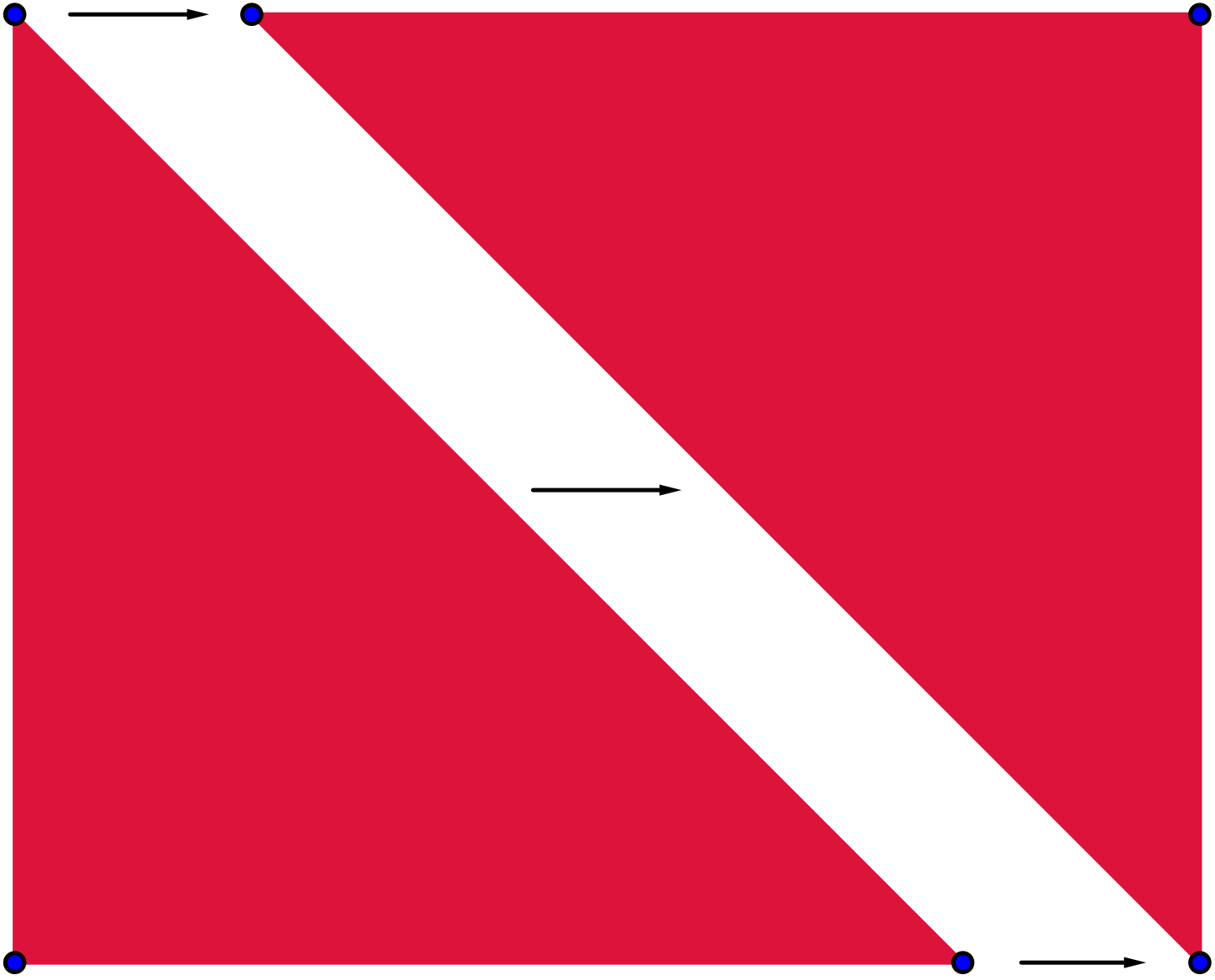


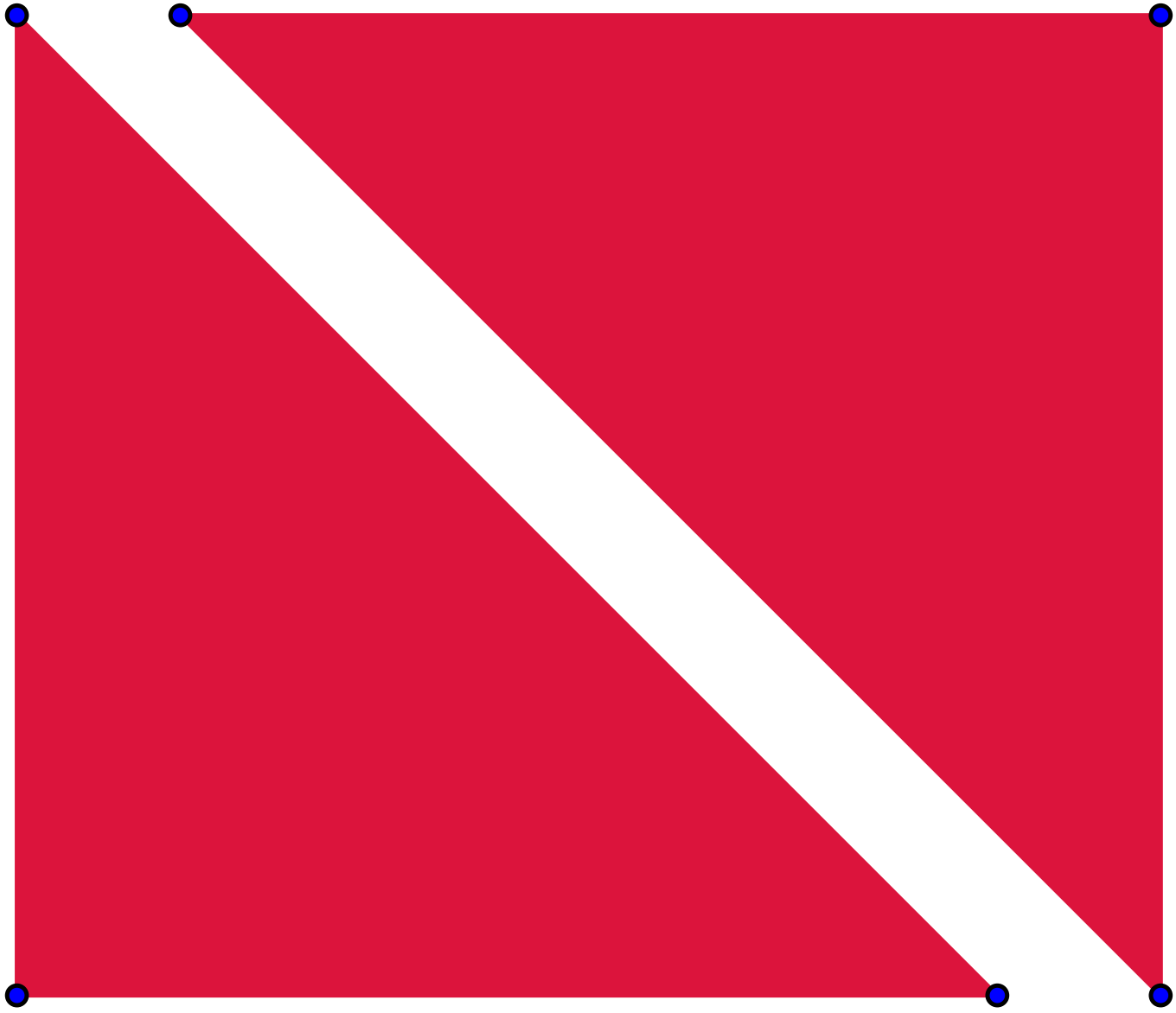


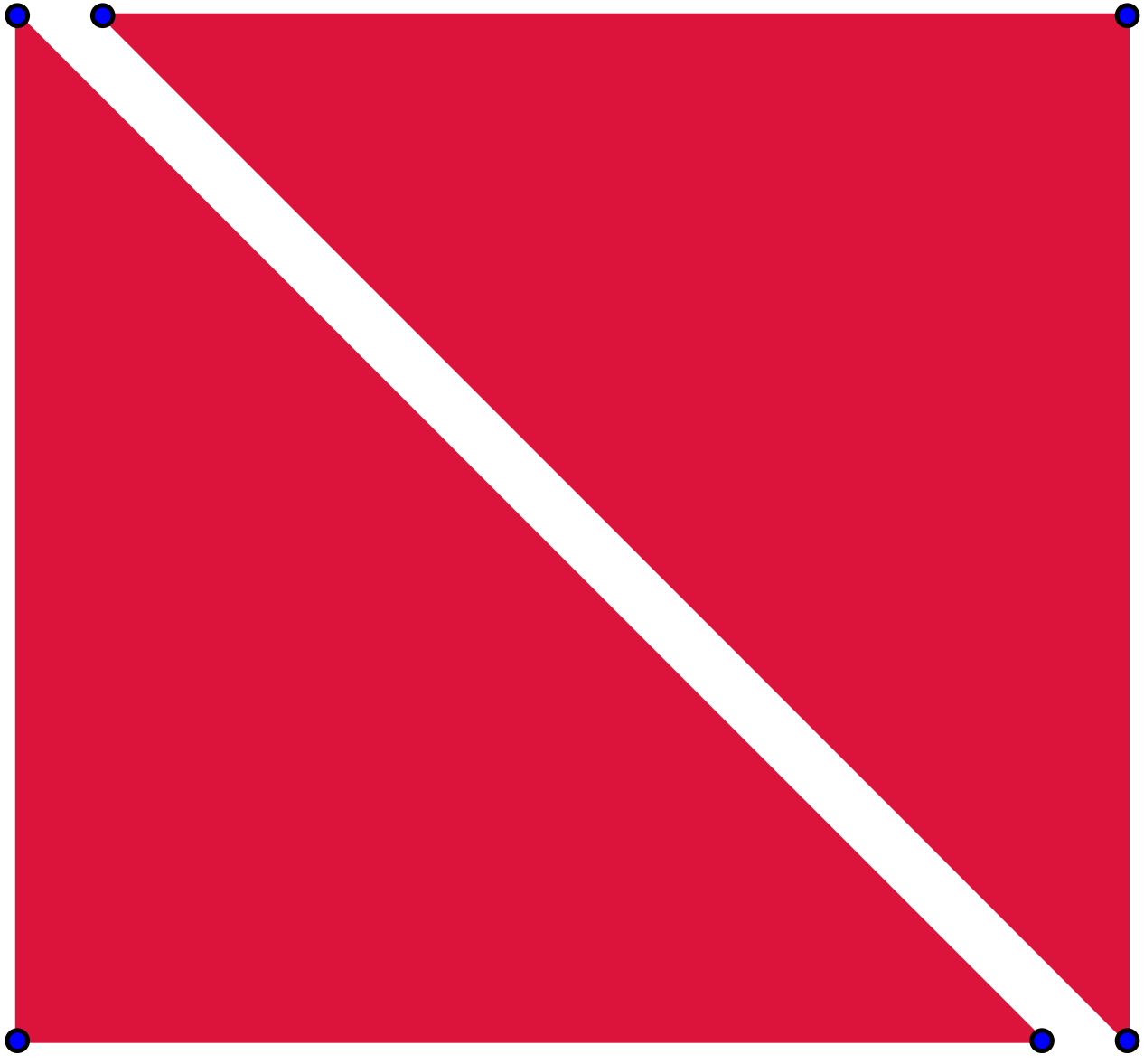


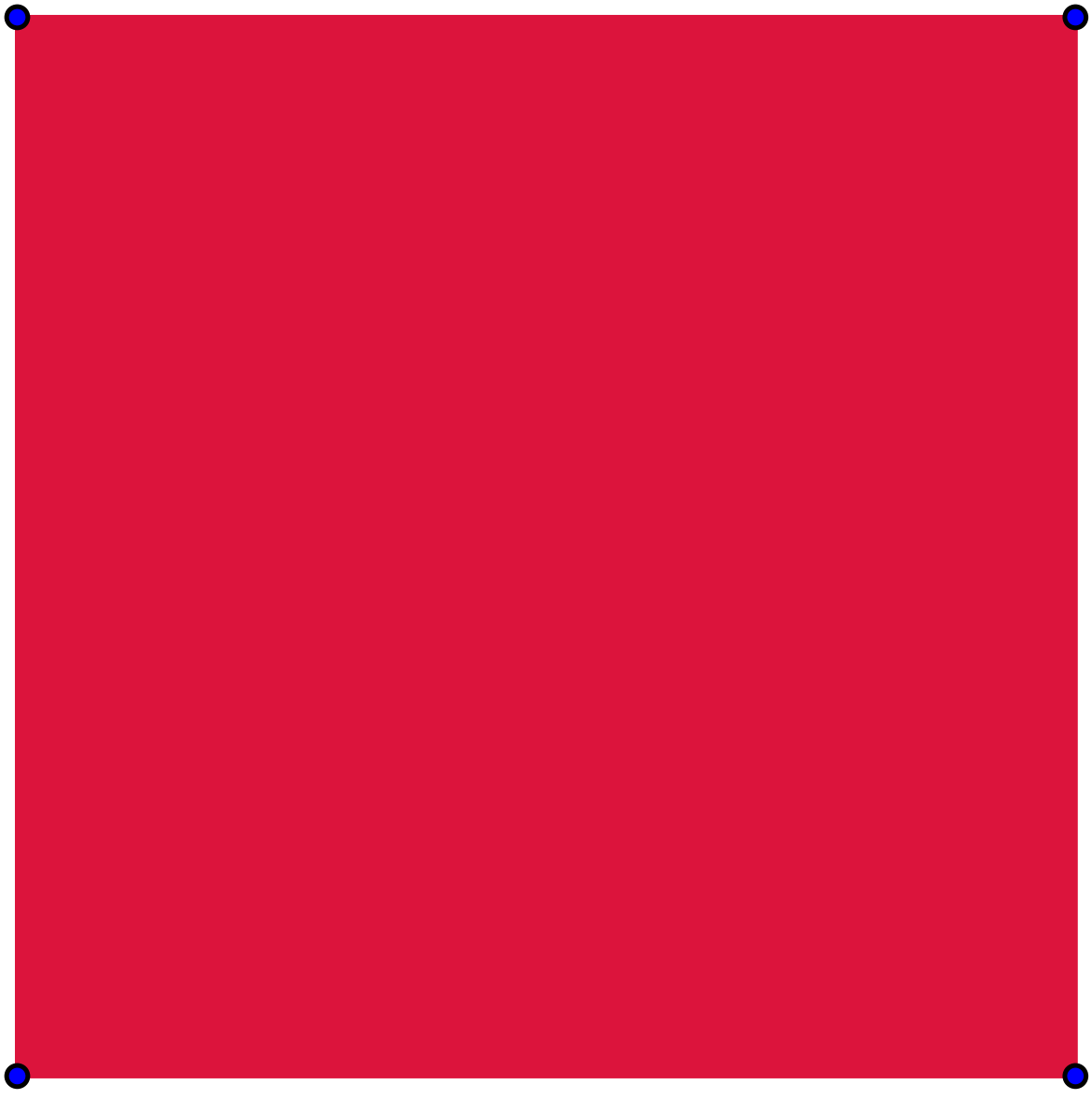


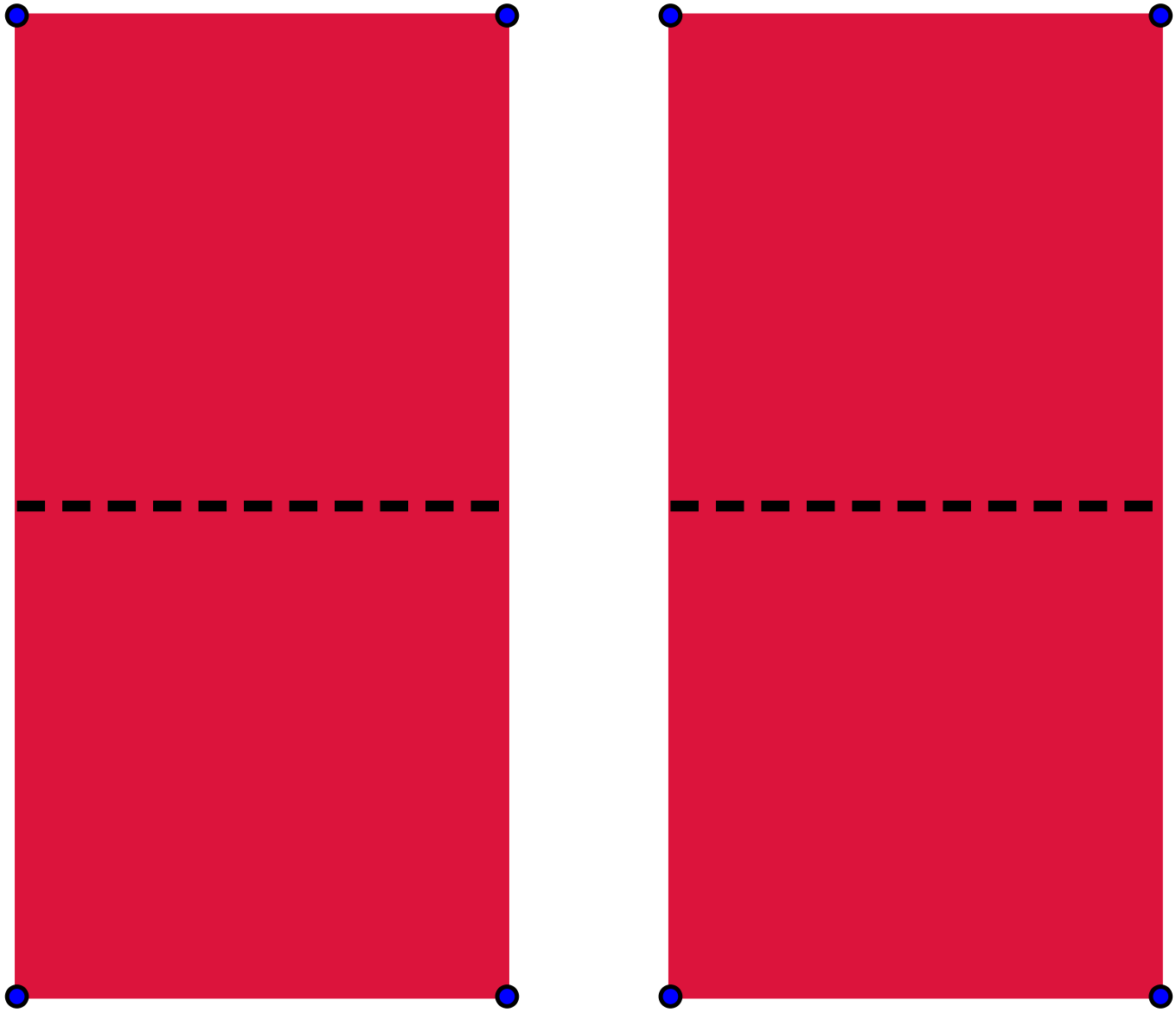


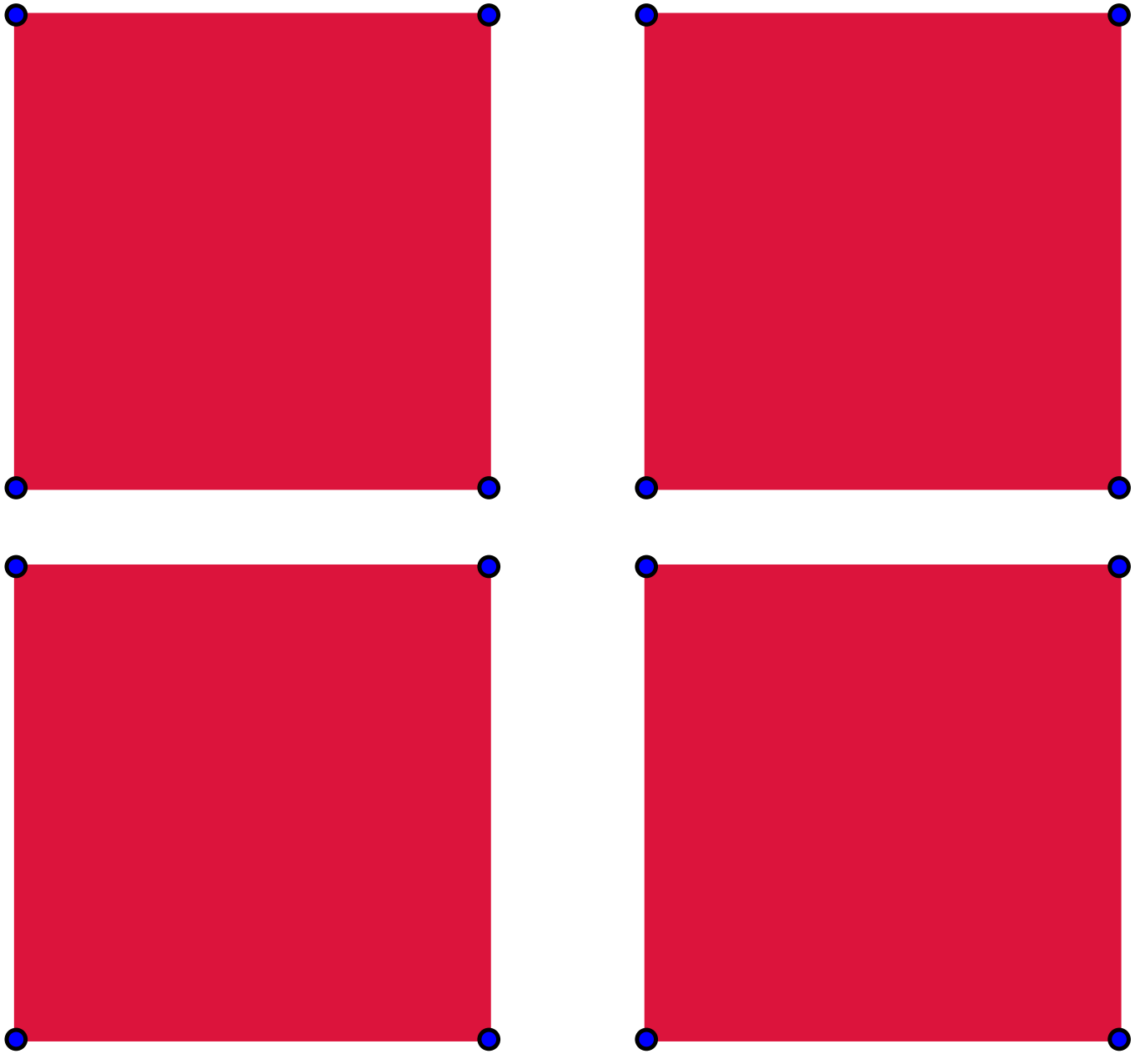


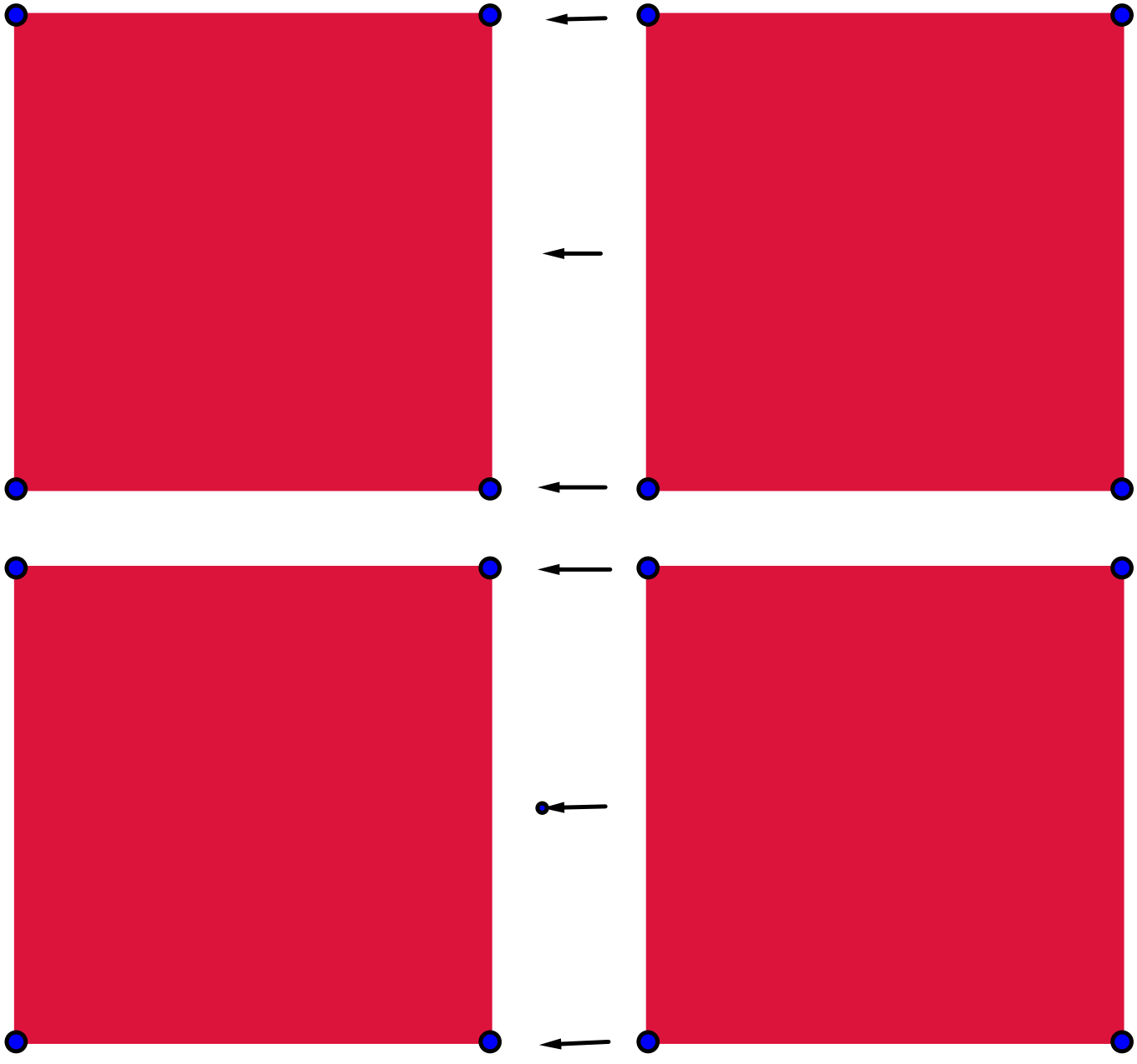


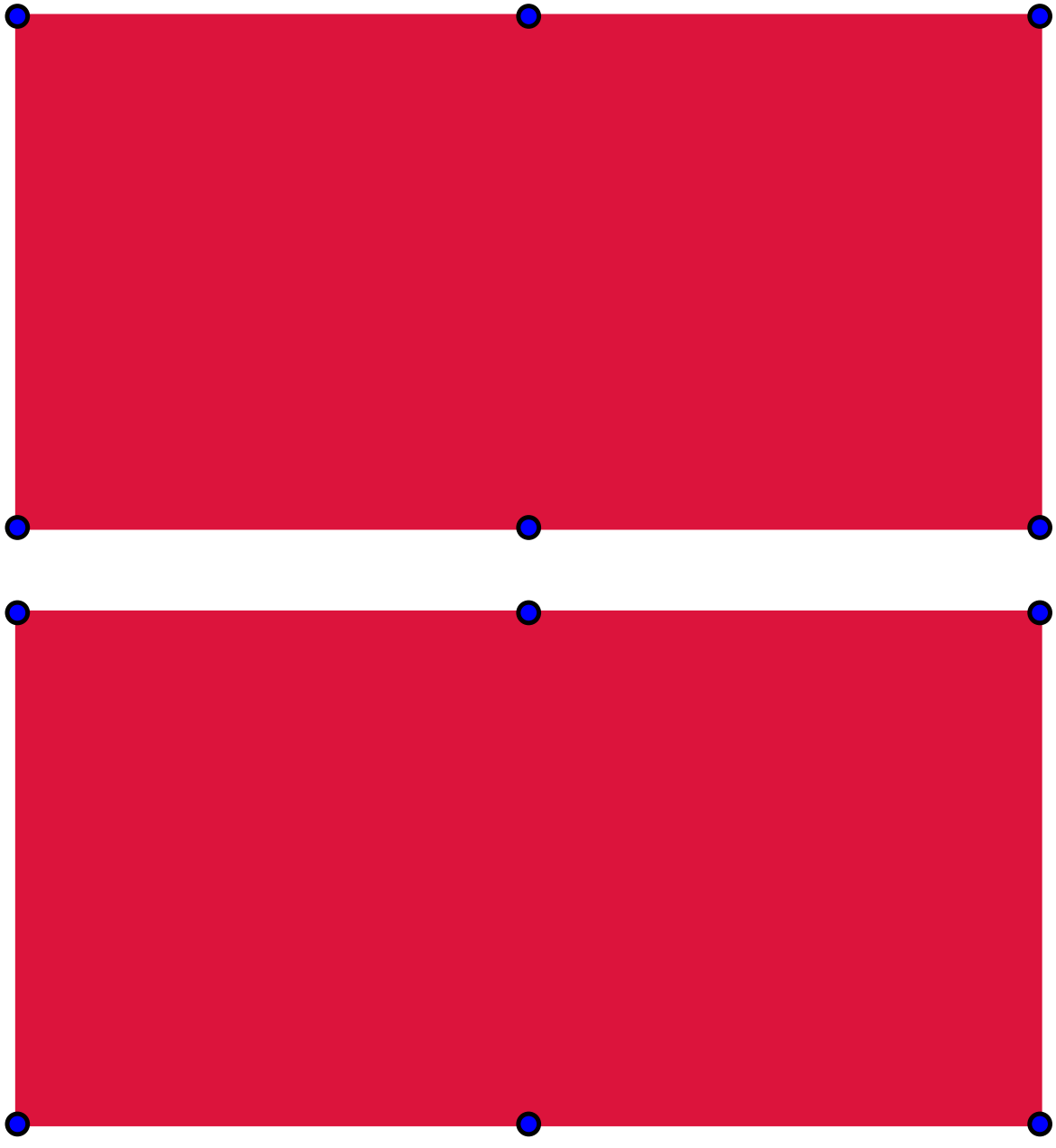


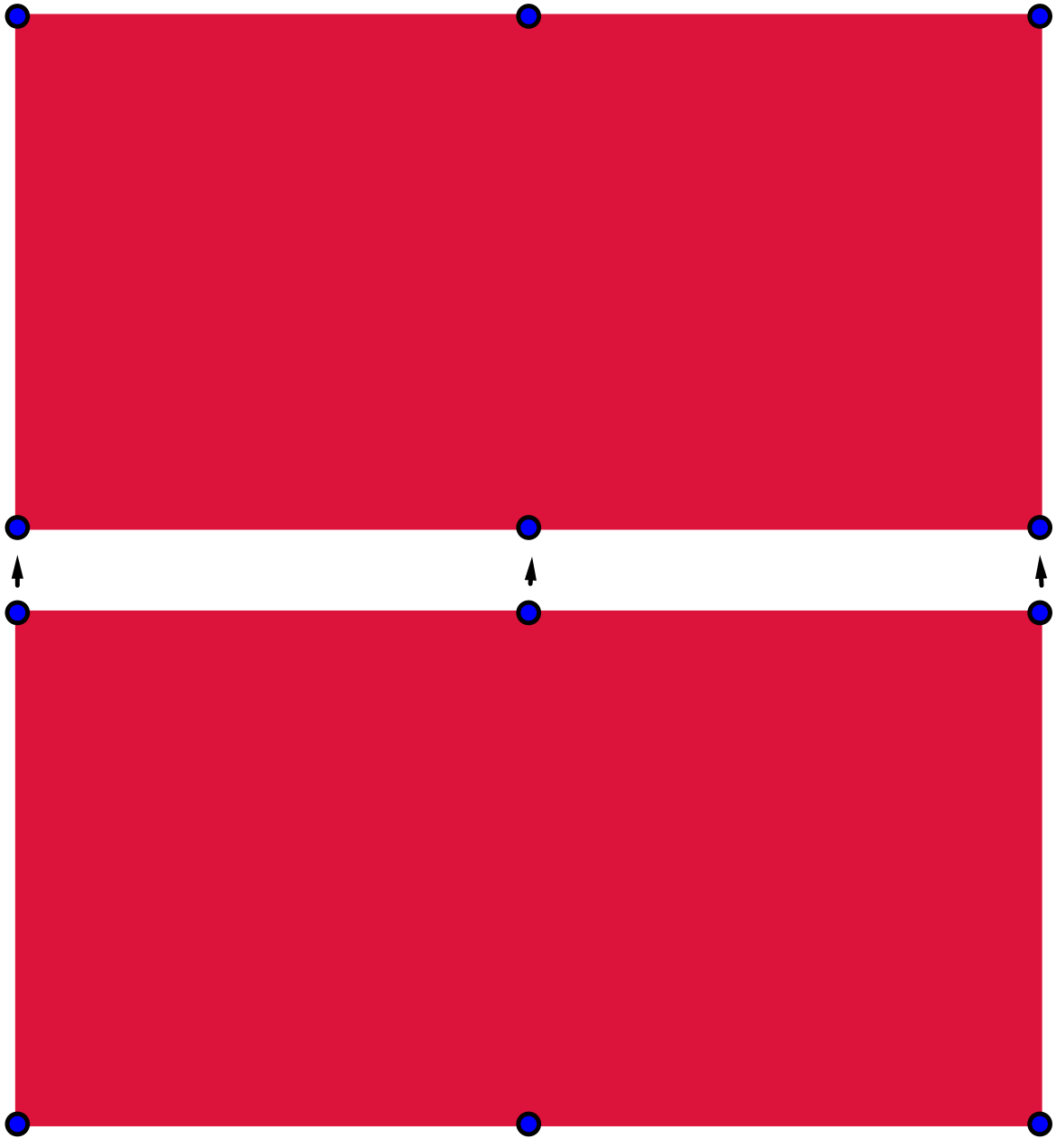


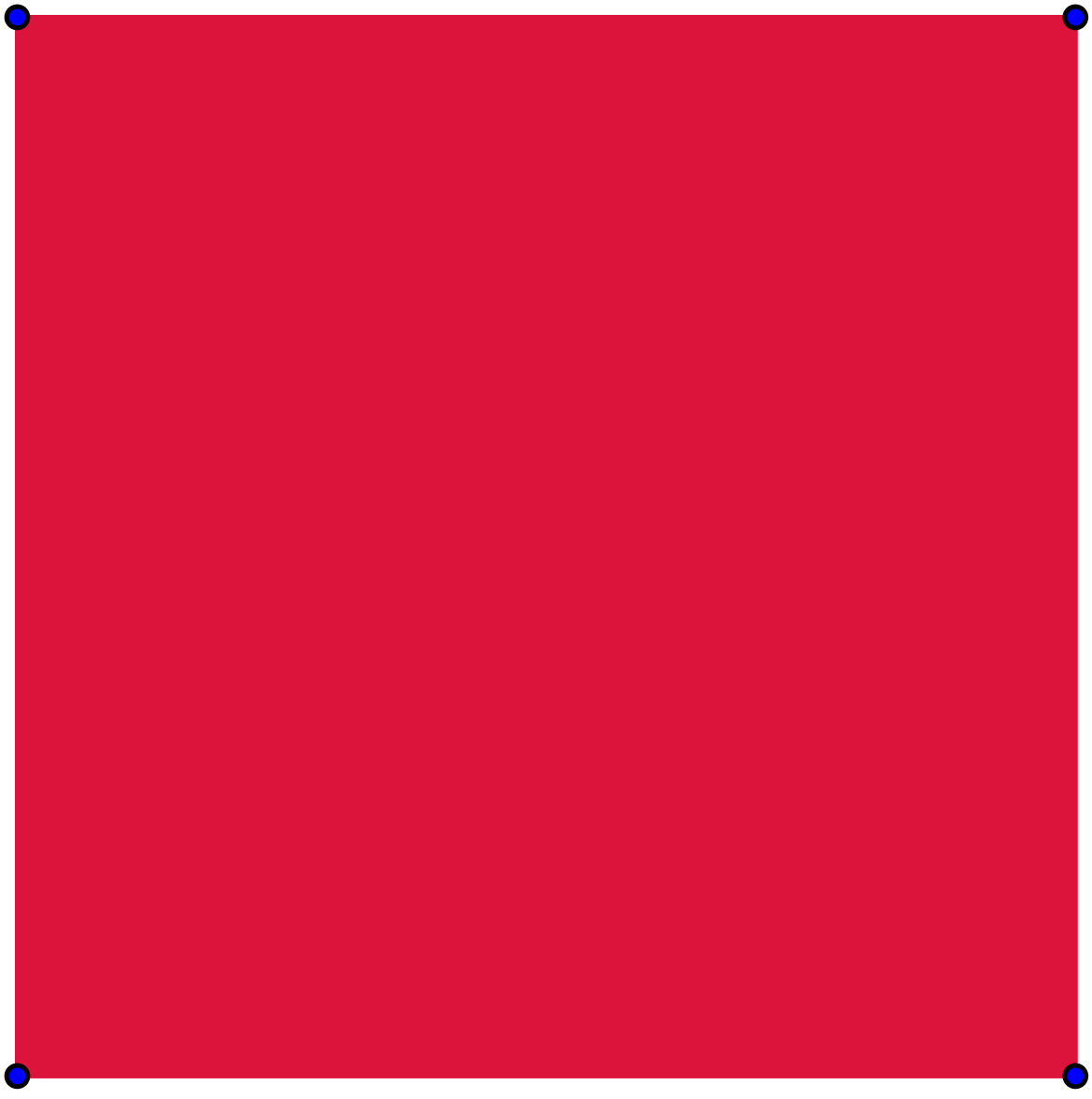




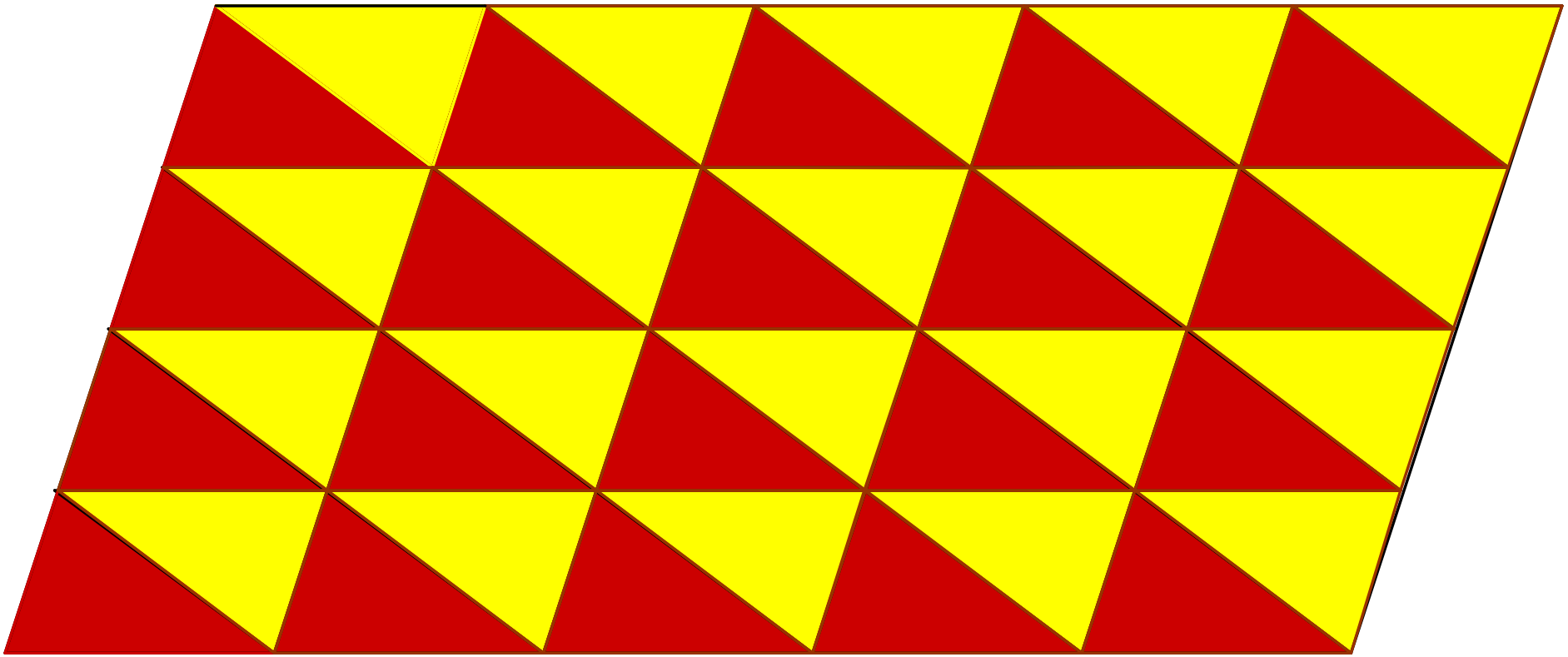




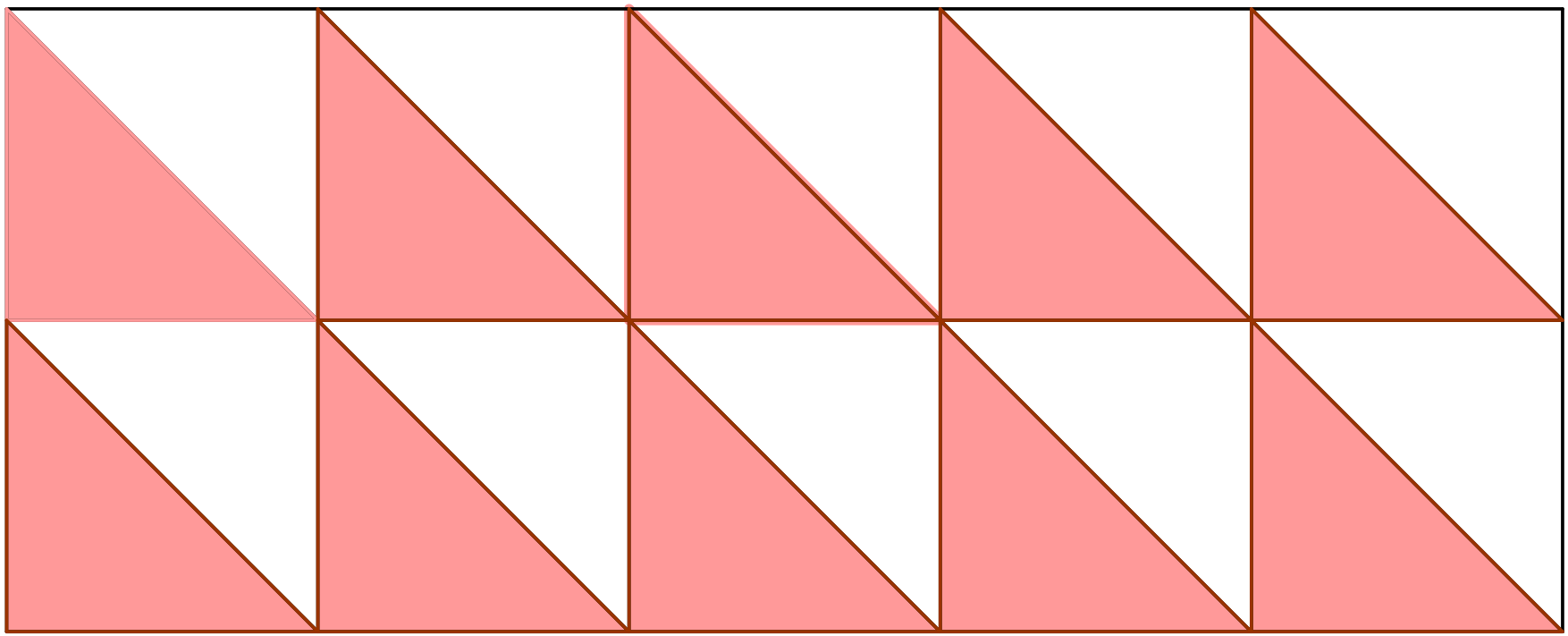




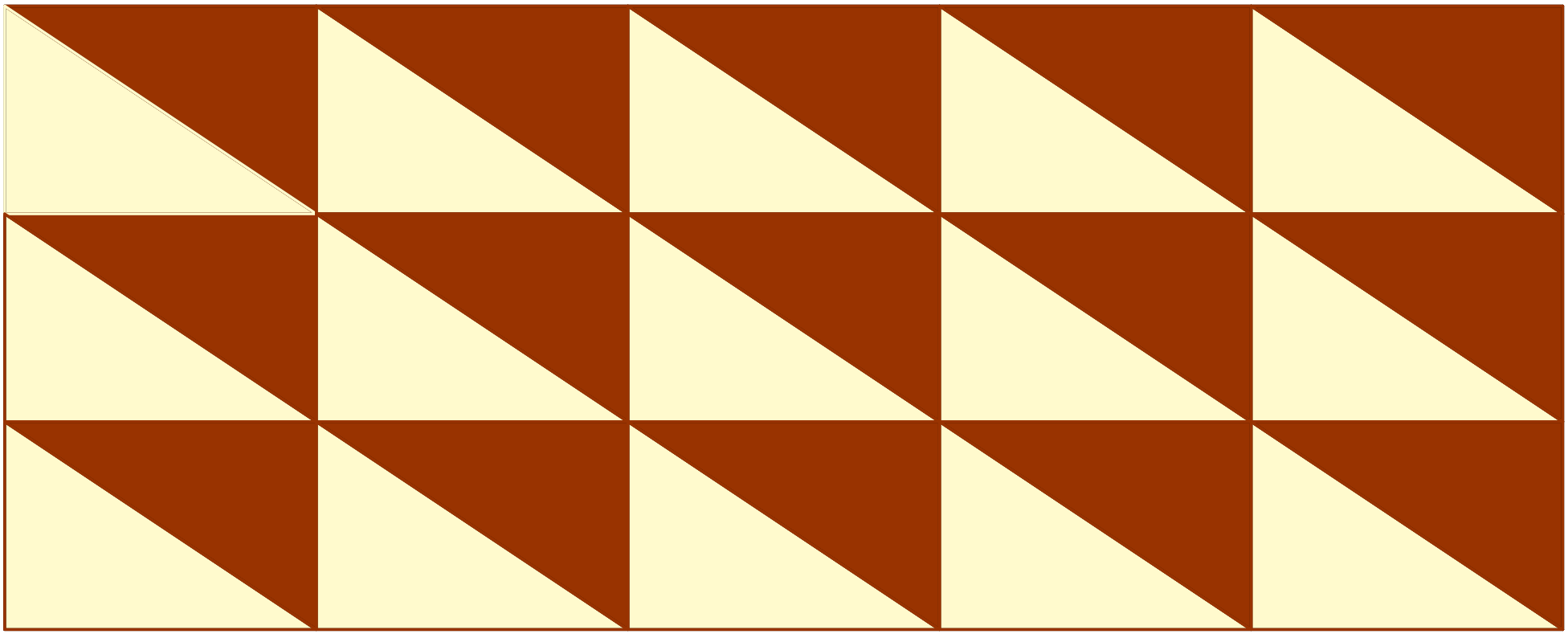
Triangle isocèle



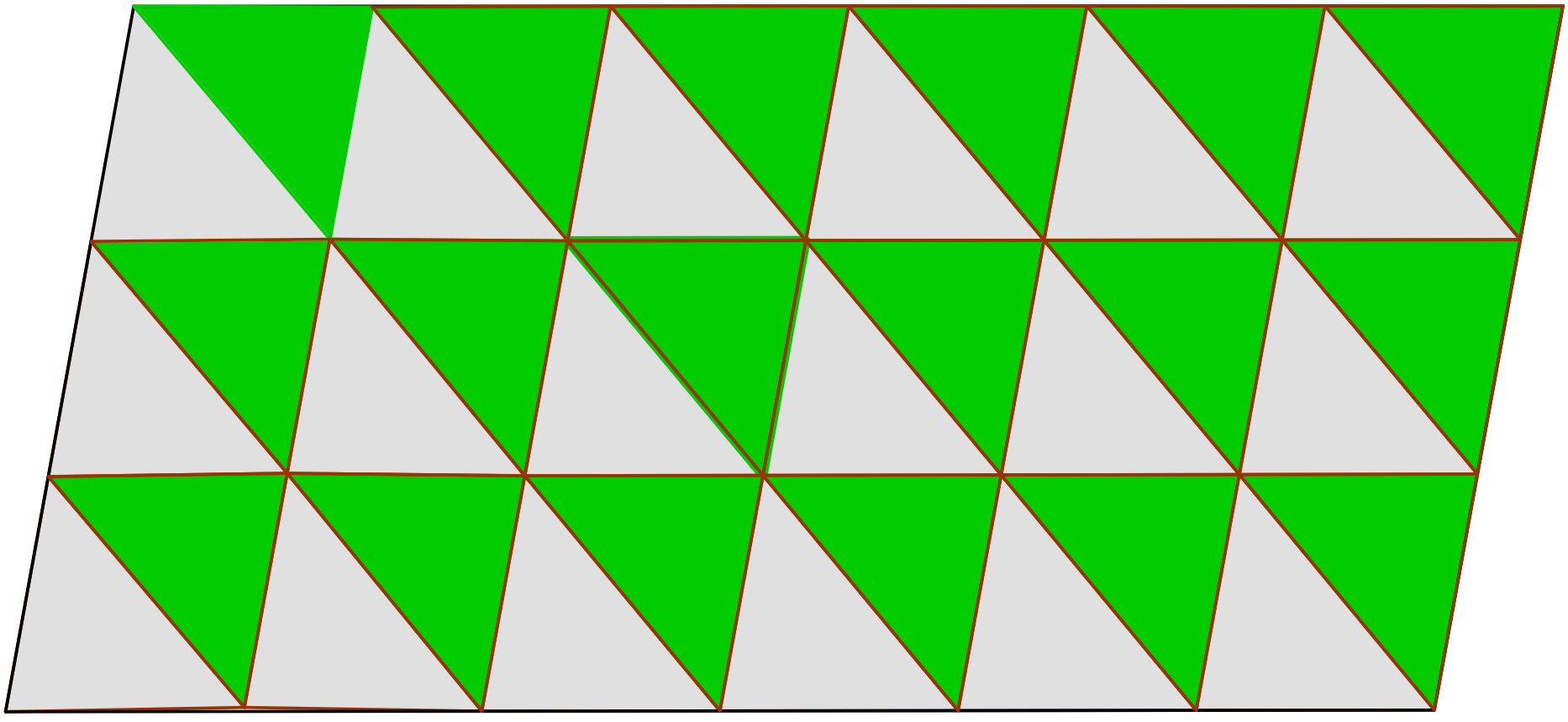
Triangle isocèle rectangle



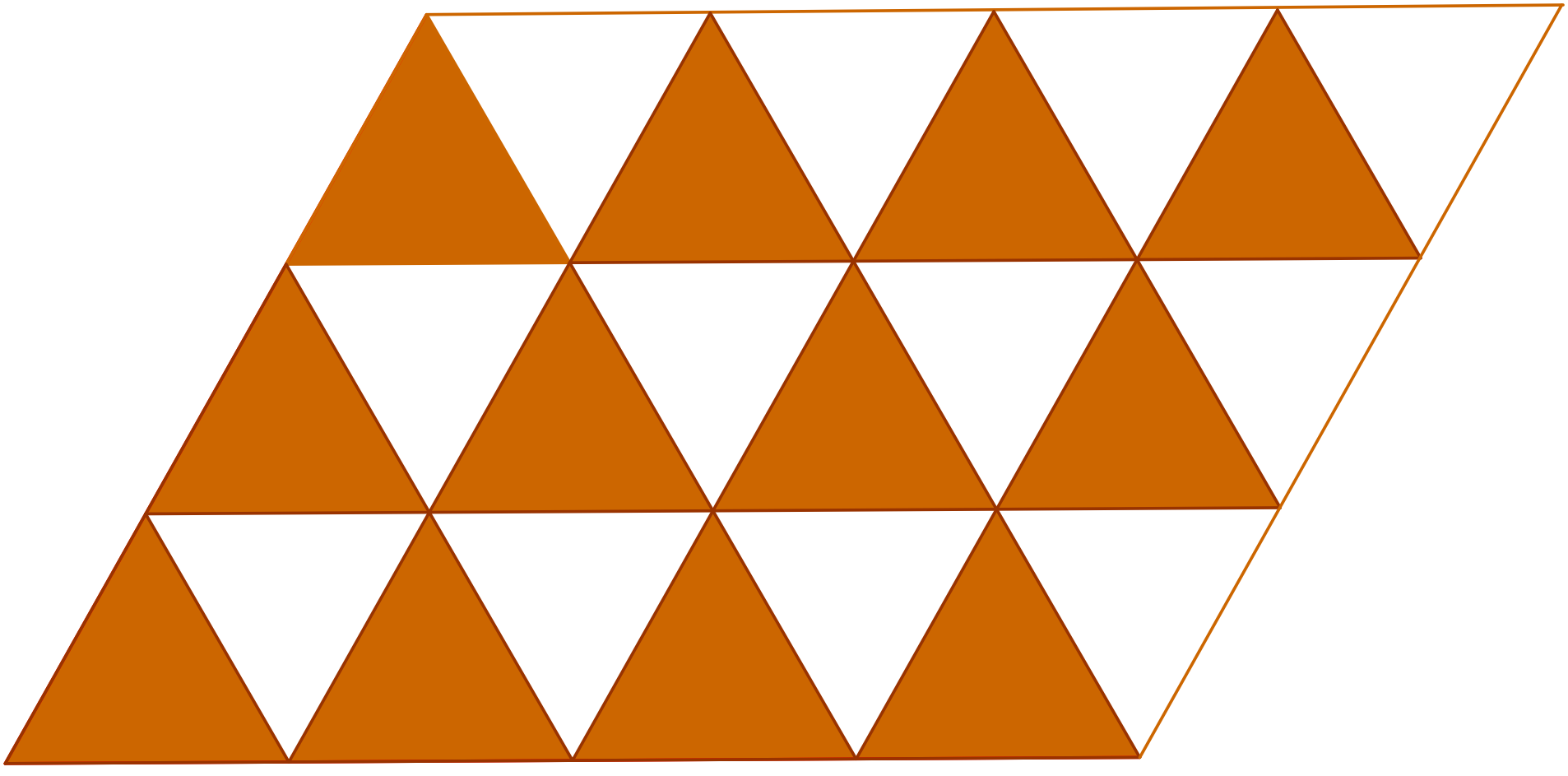
Triangle rectangle

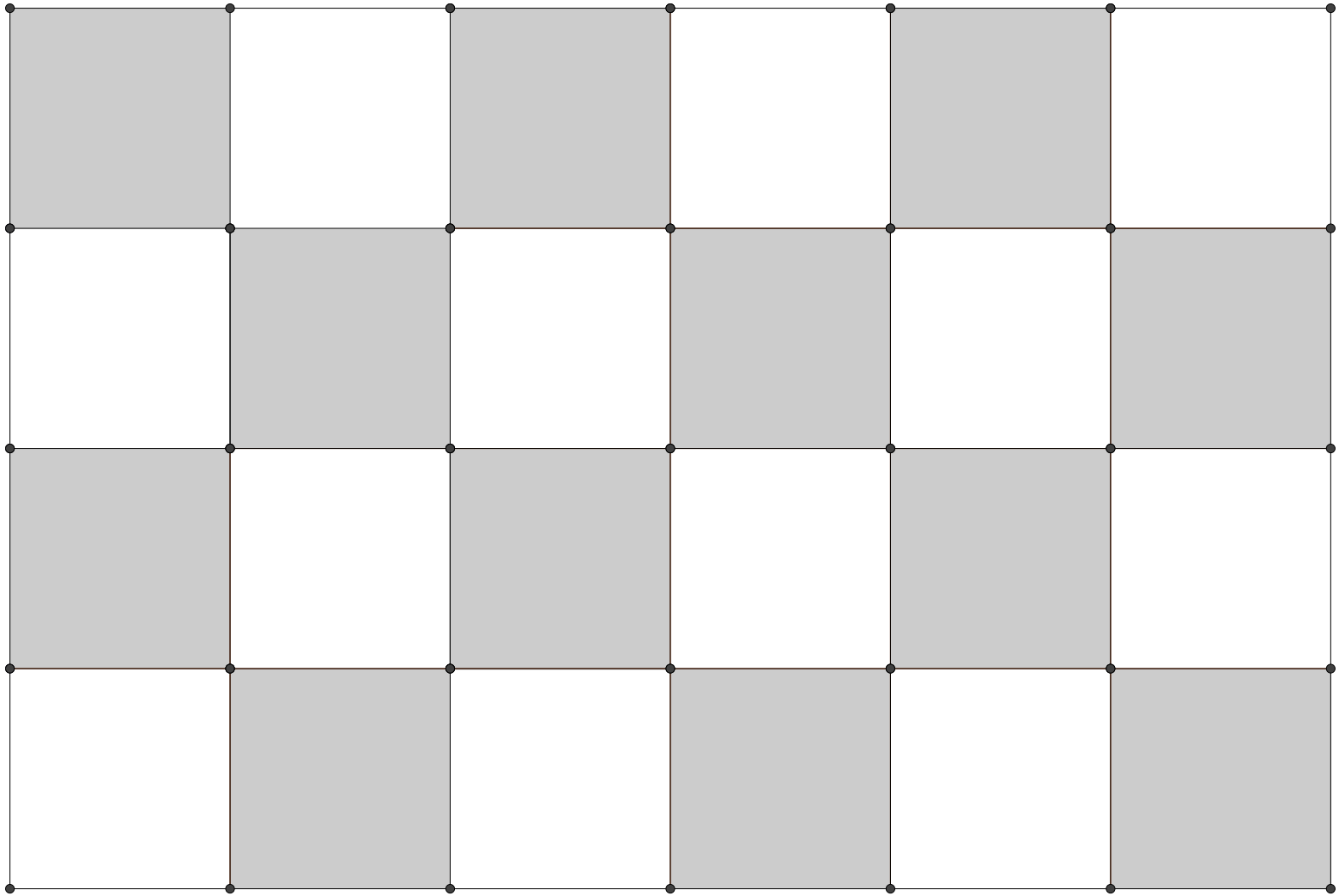


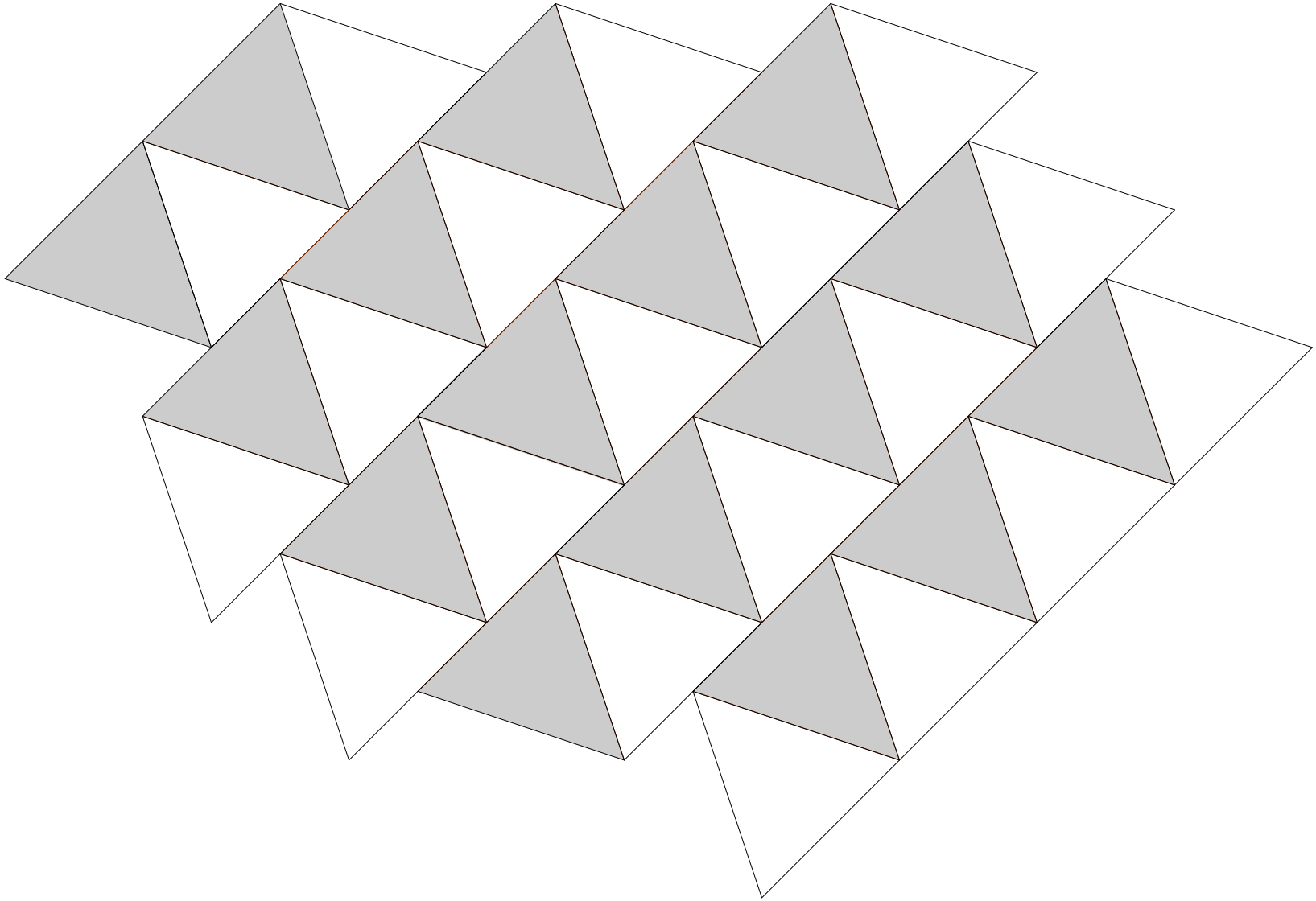
Triangle quelconque



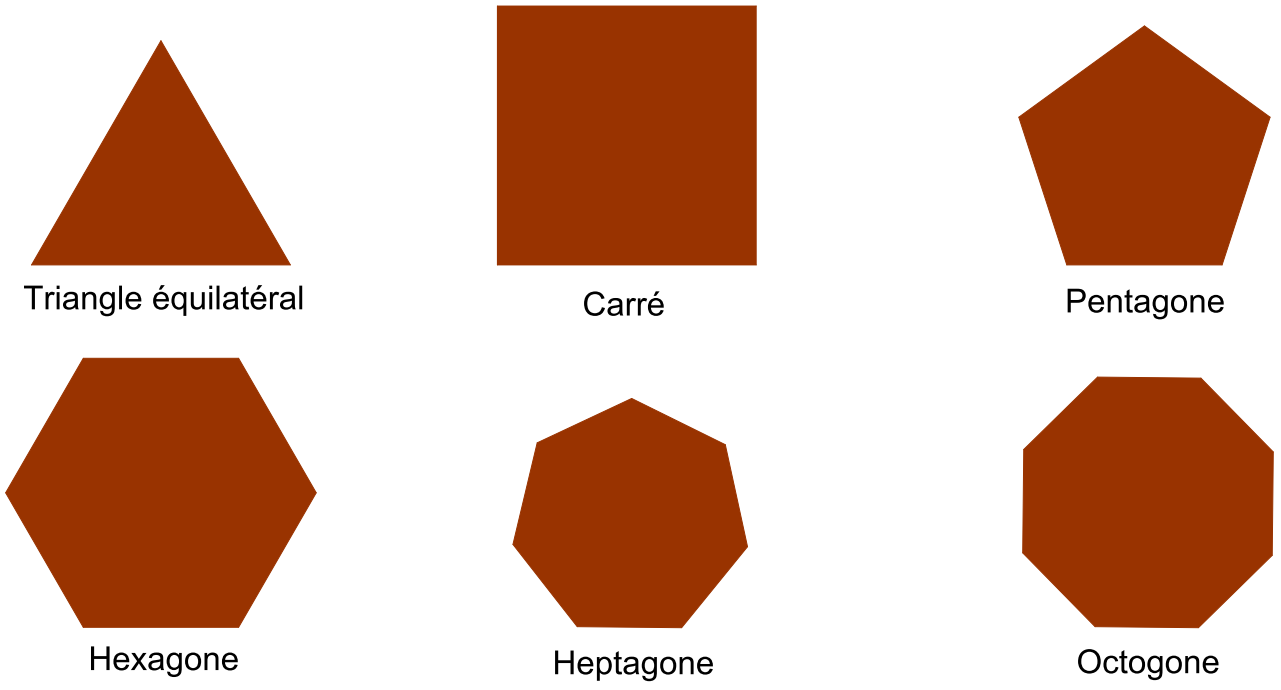
Triangle équilatéral





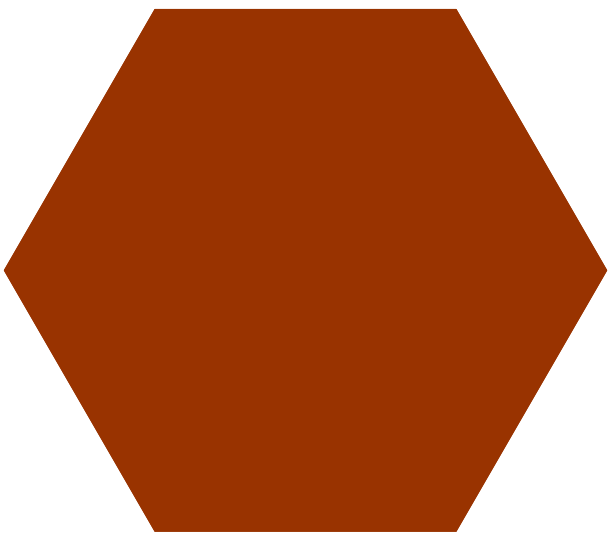


Quels sont les polygones réguliers qui pavent de façon périodique le plan euclidien $\mathbb{E}$?

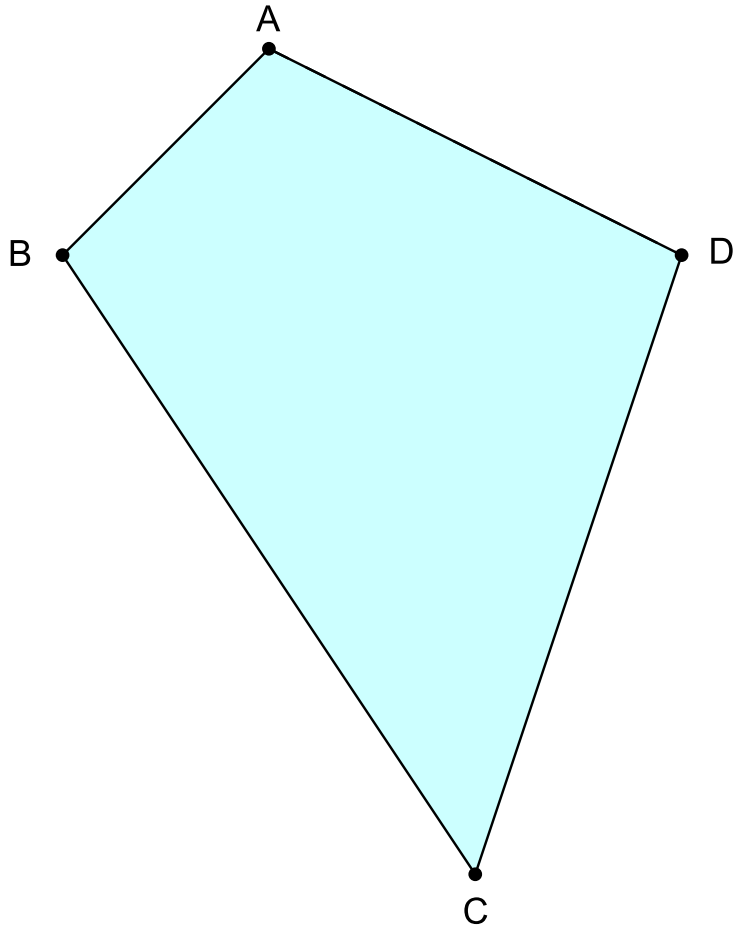


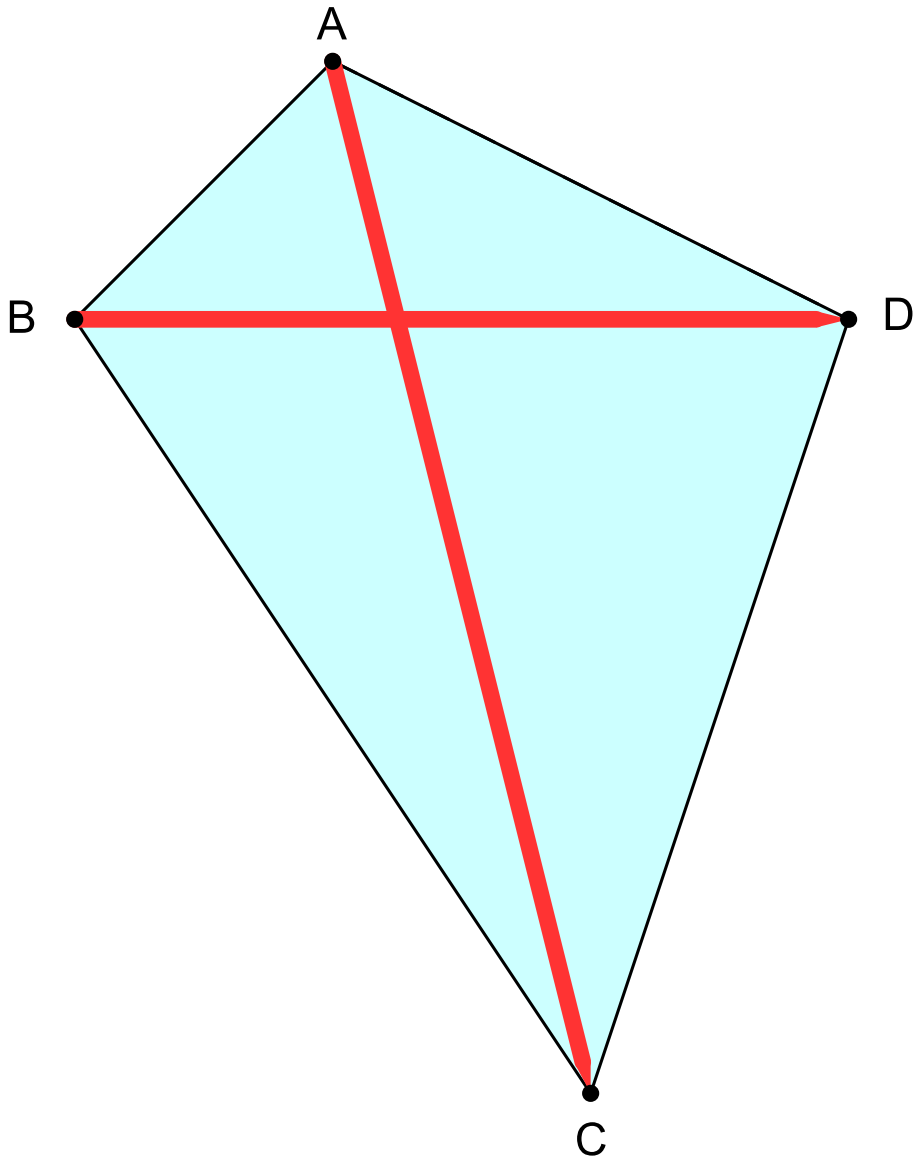
Polygones réguliers

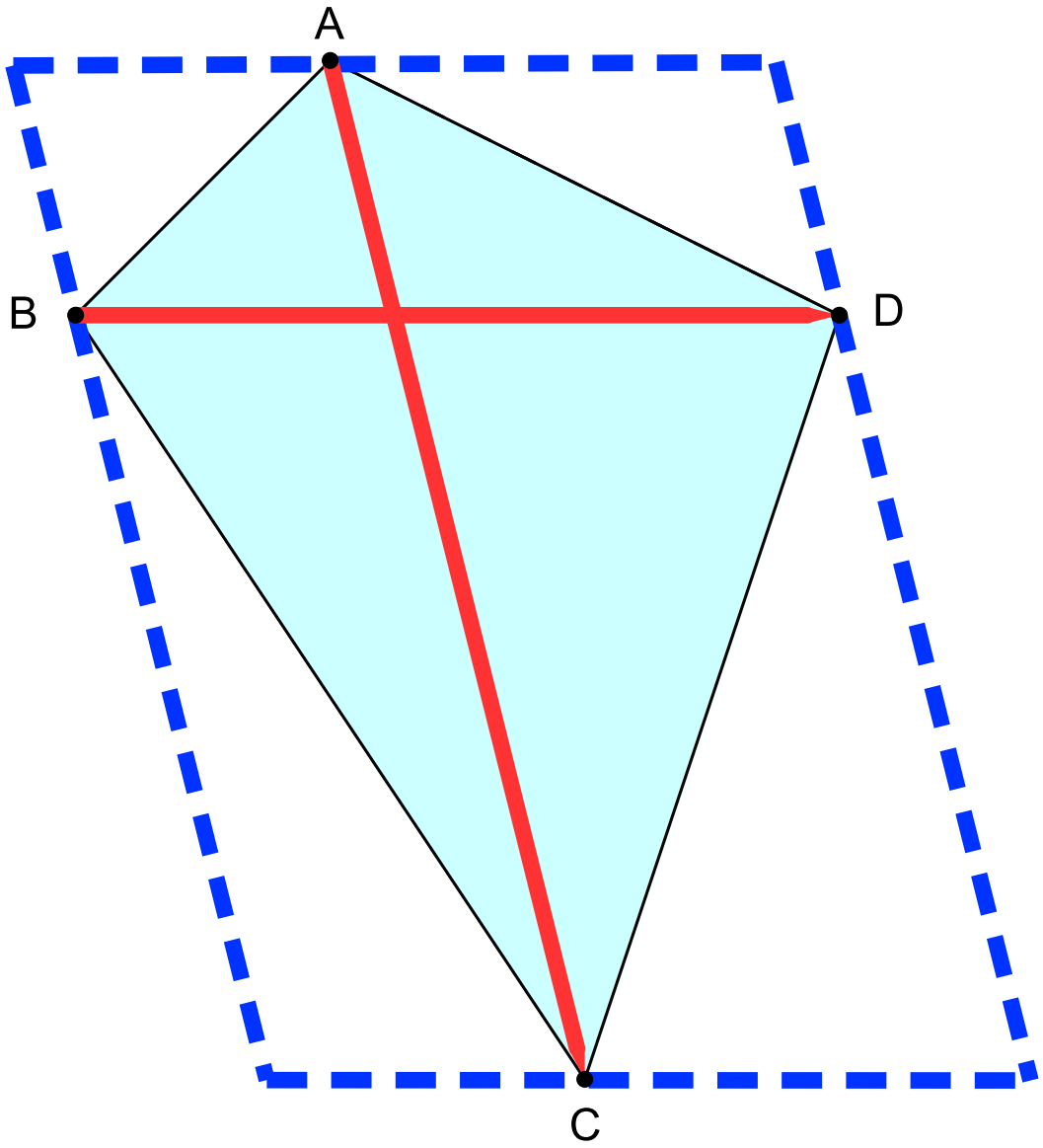
Polygones réguliers paveurs

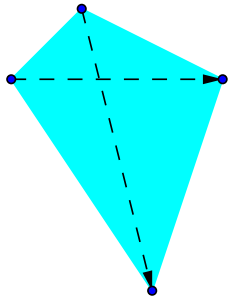


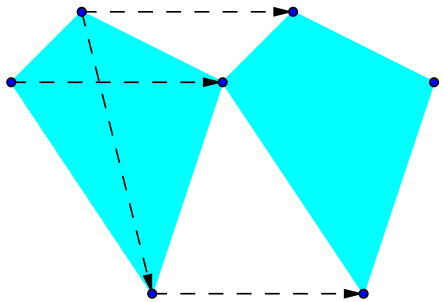
Carrelage par quadrilatères

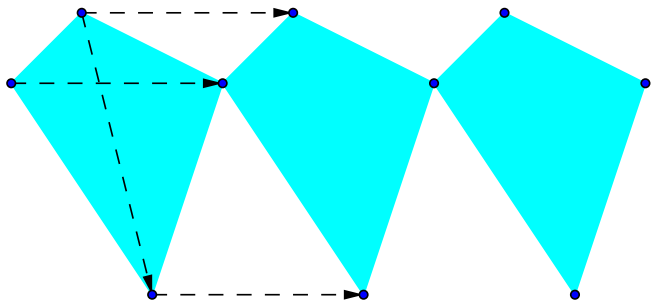


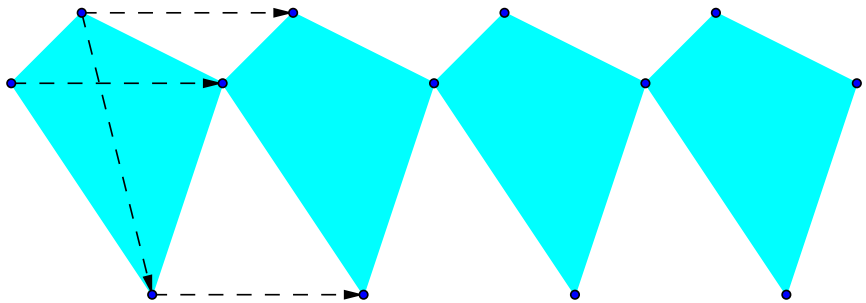

A
B
D
C

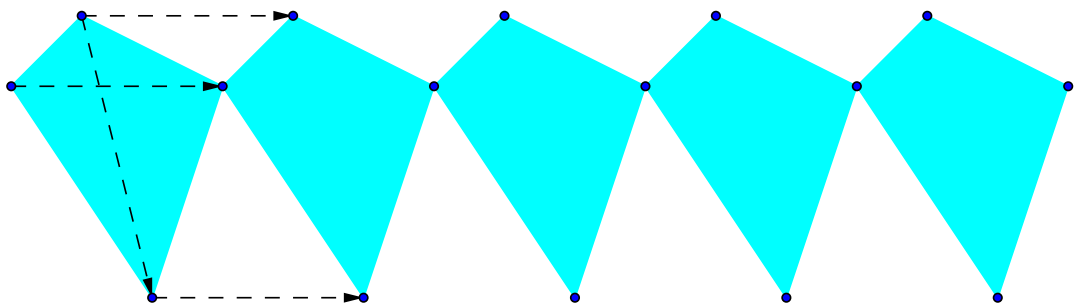

A
B
D
C

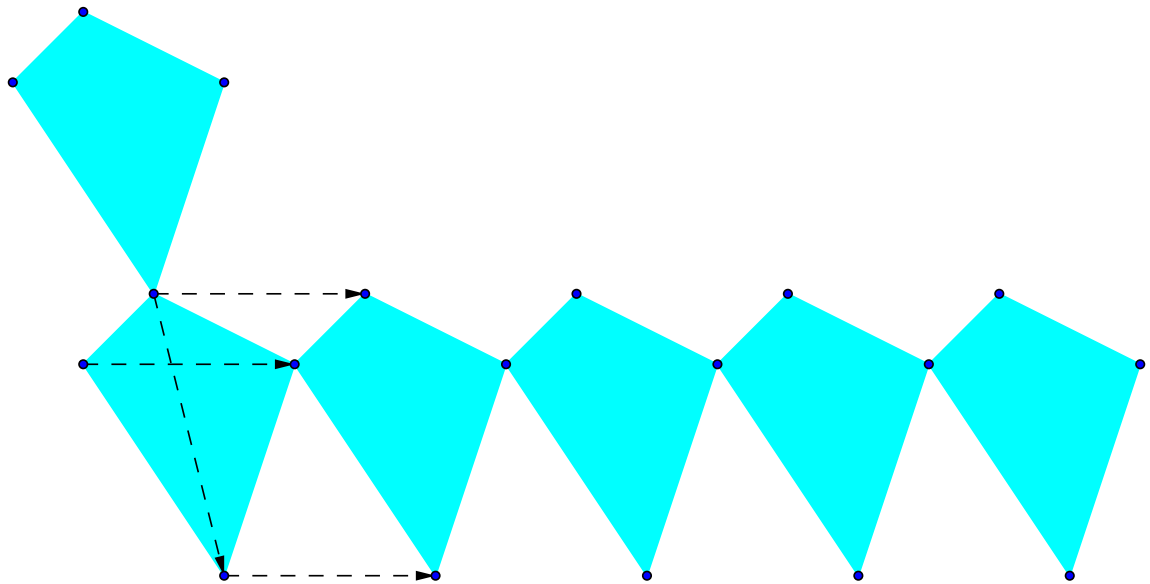


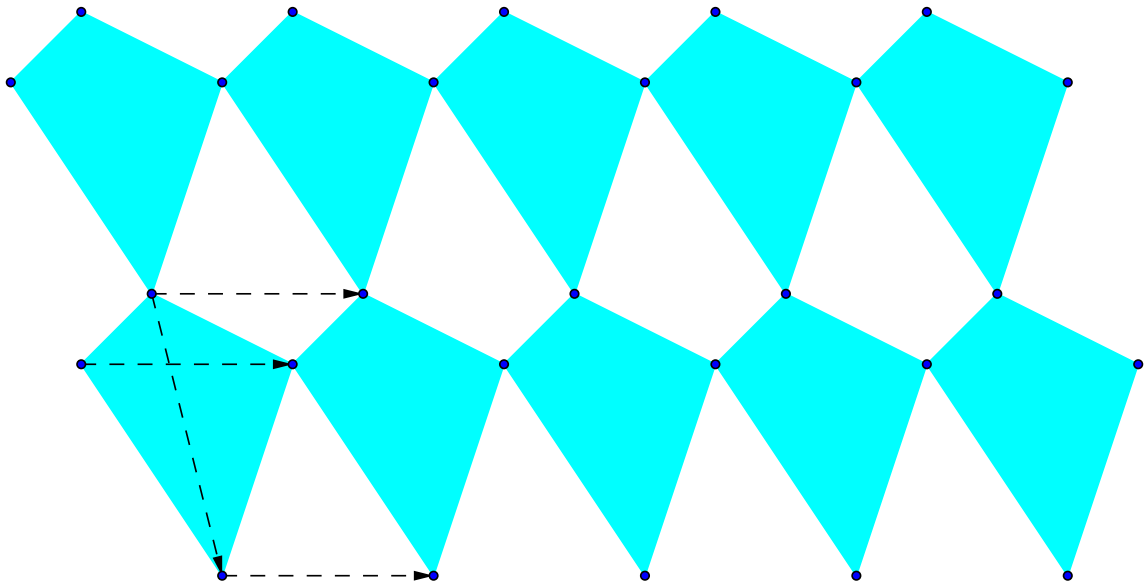


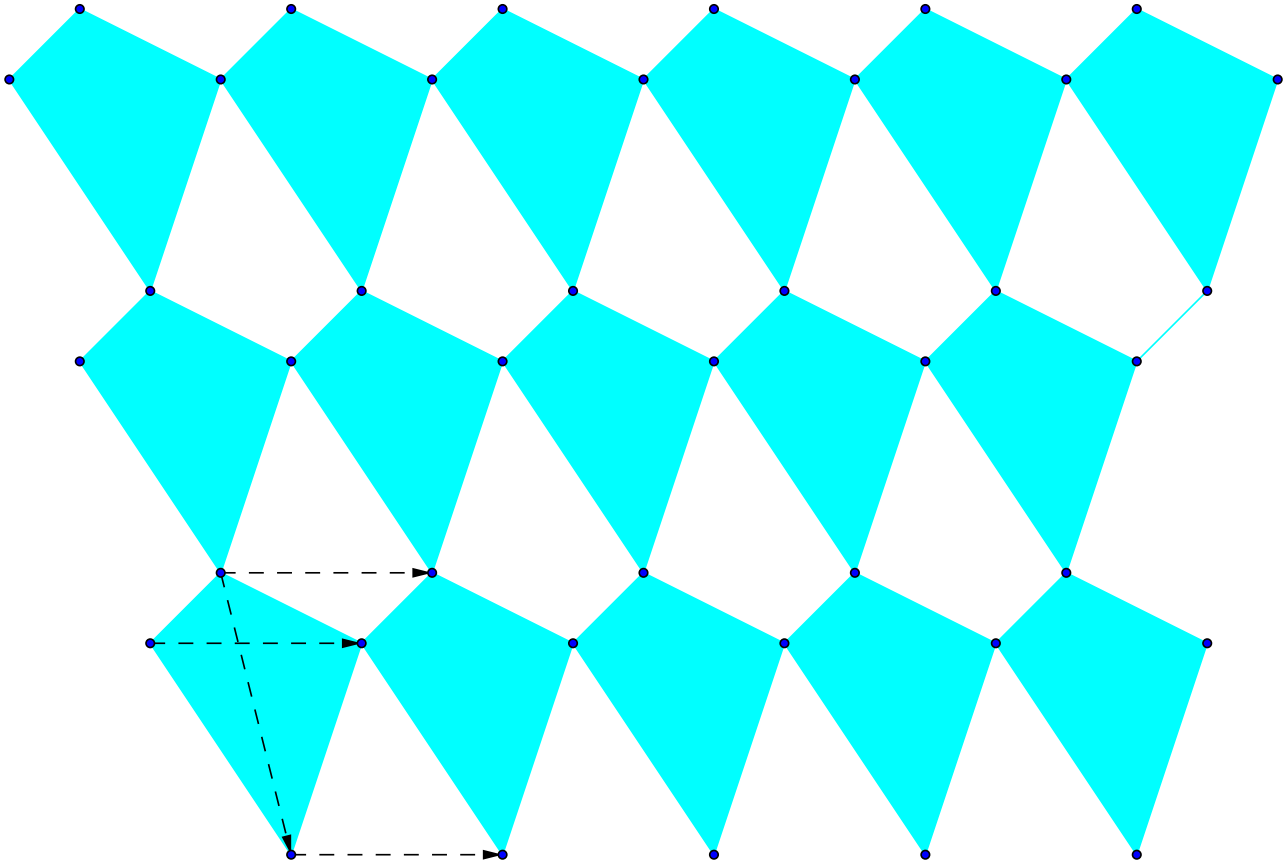


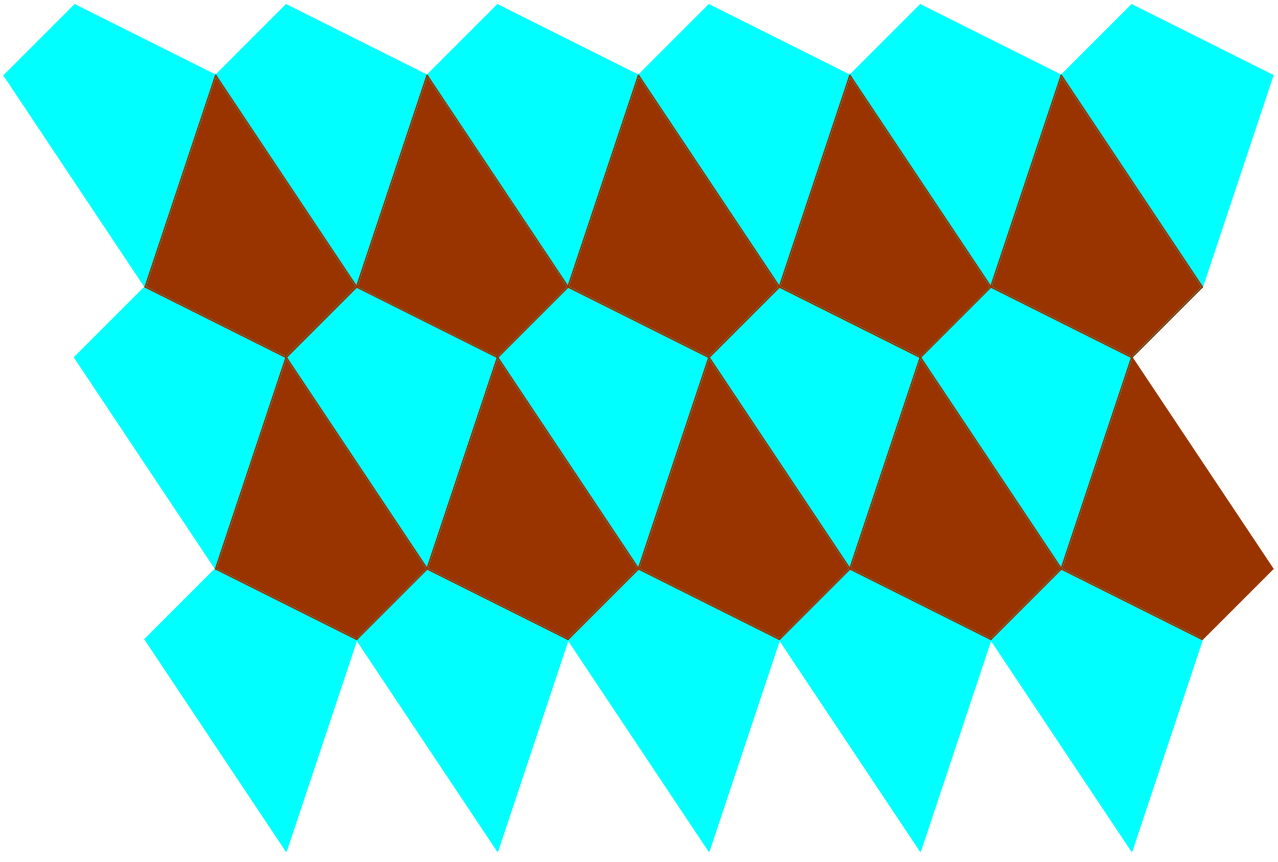


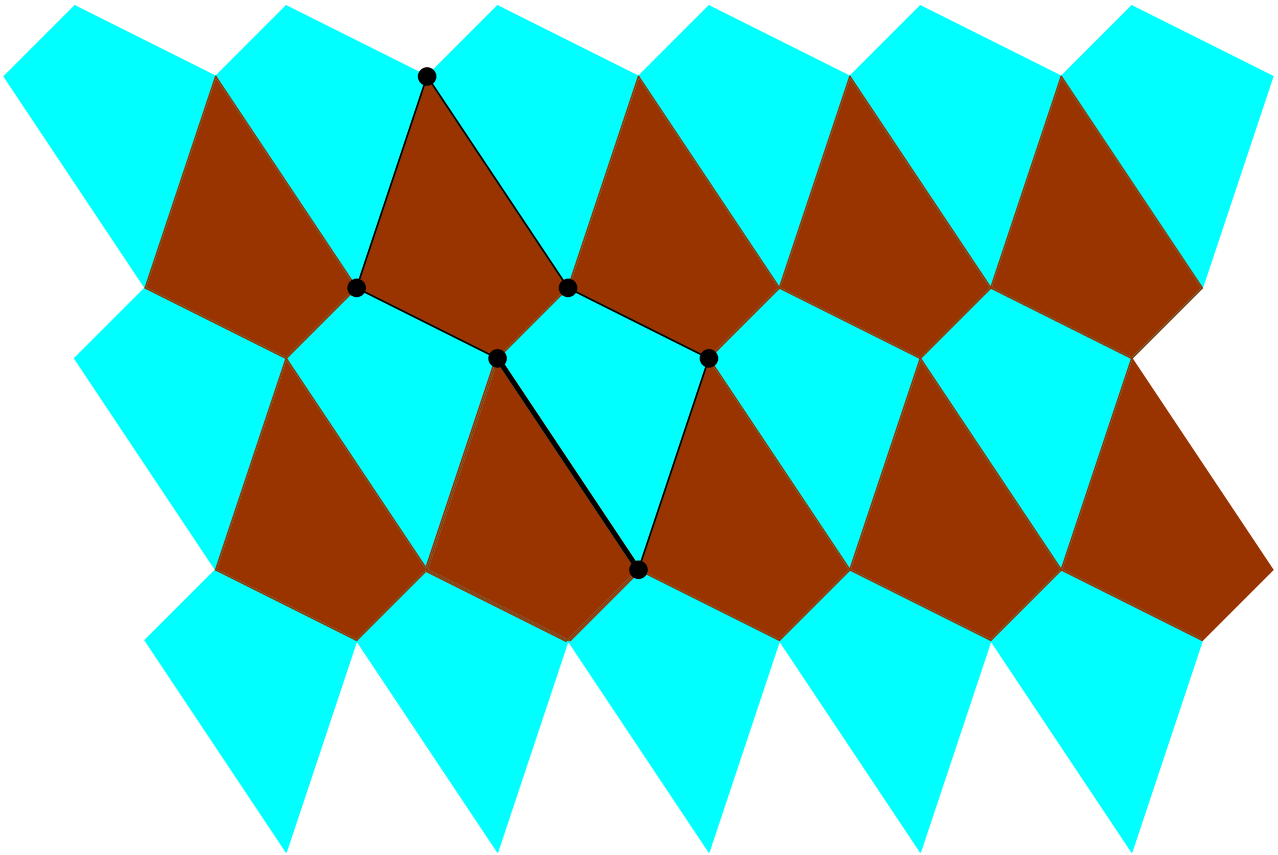


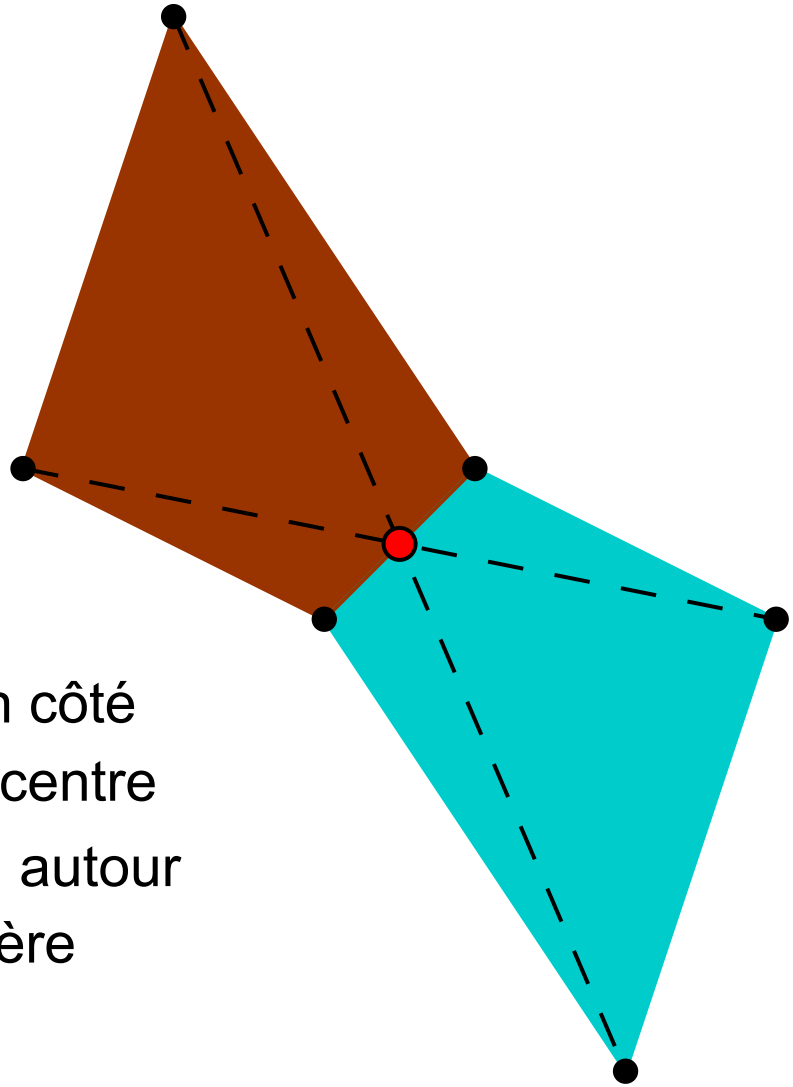




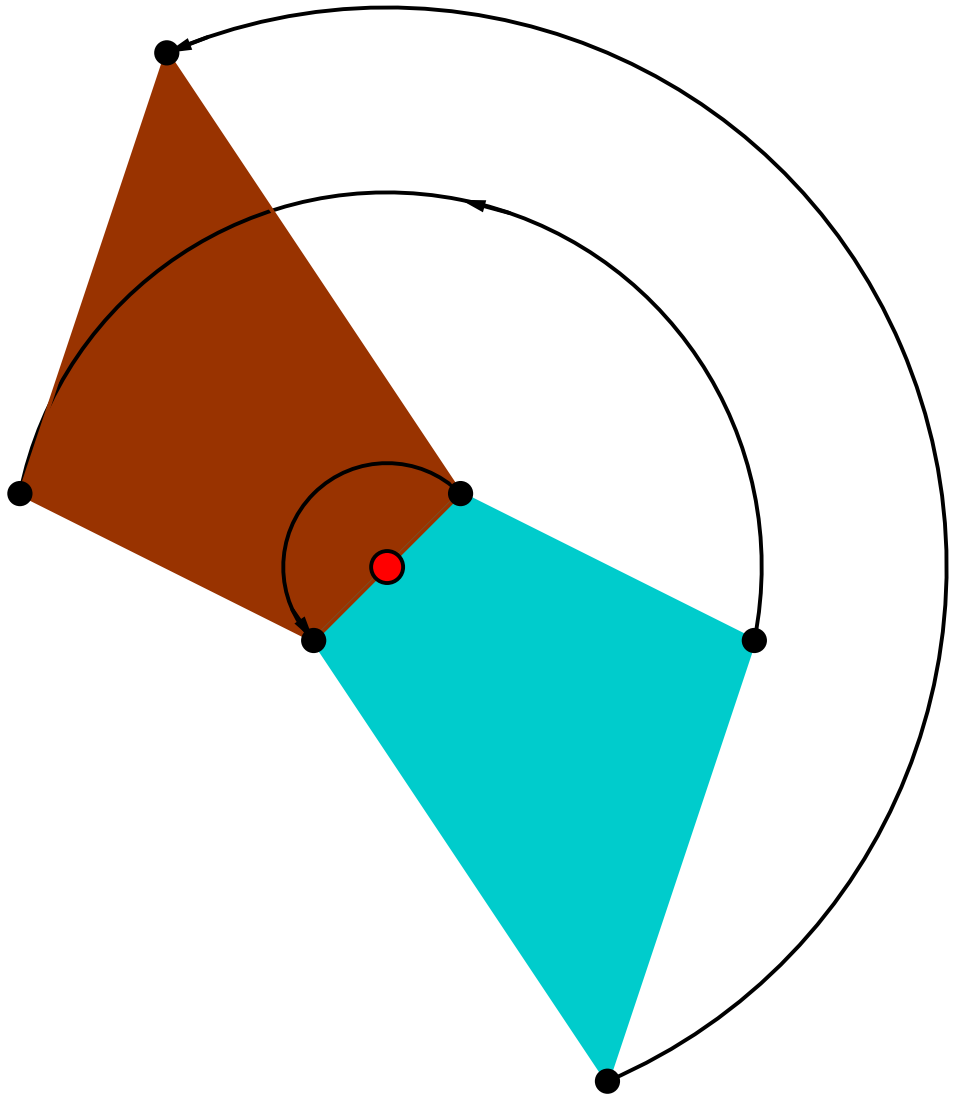






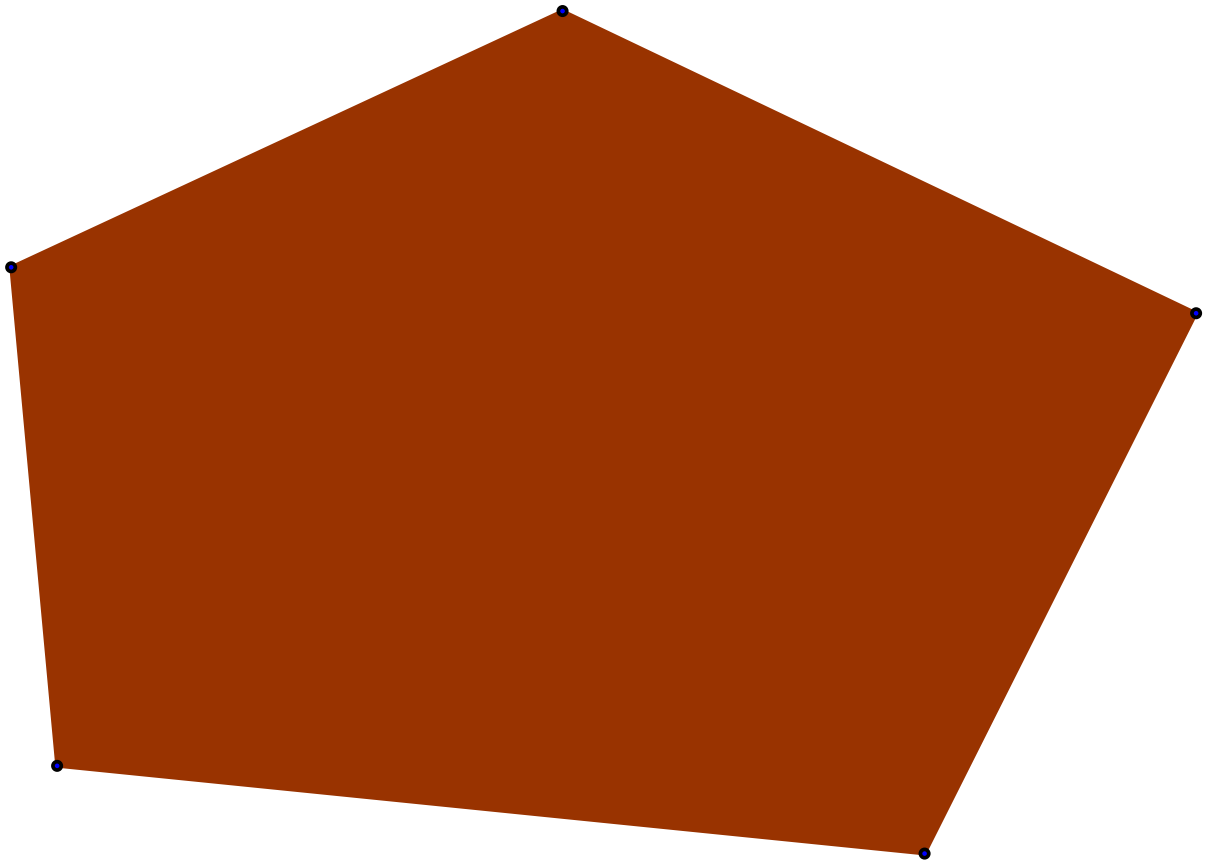


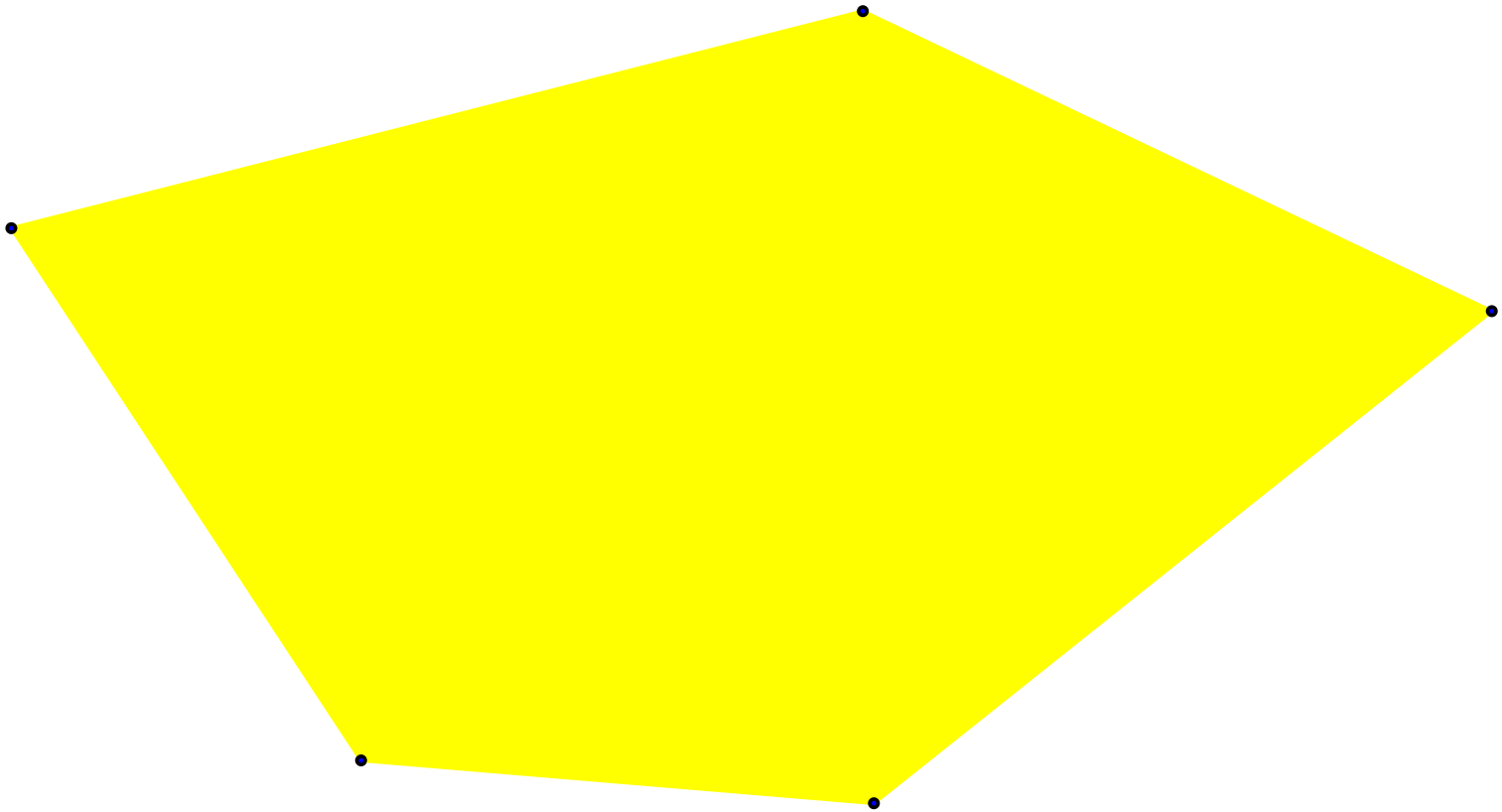
Deux quadrilatères collés sur un côté donnent un hexagone ayant un centre de symétrie. La rotation de 180° autour de ce centre amène le quadrilatère bleu sur le marron.

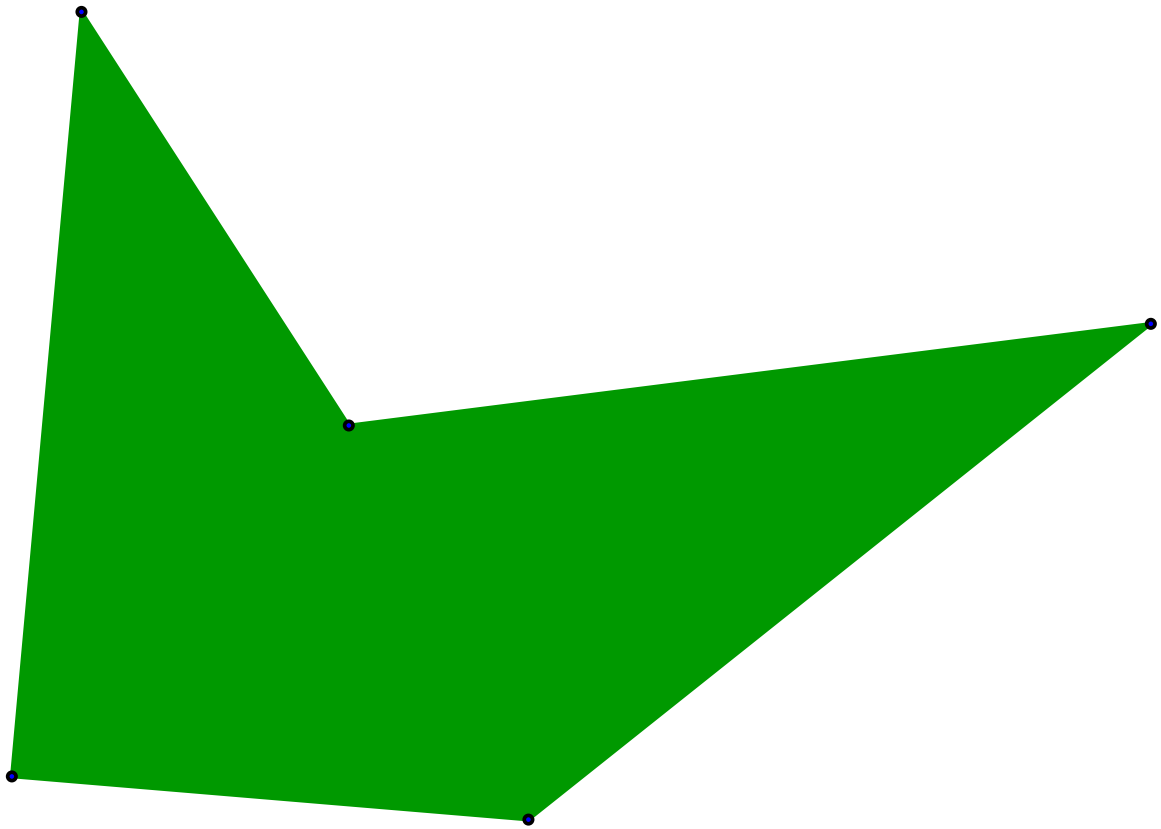


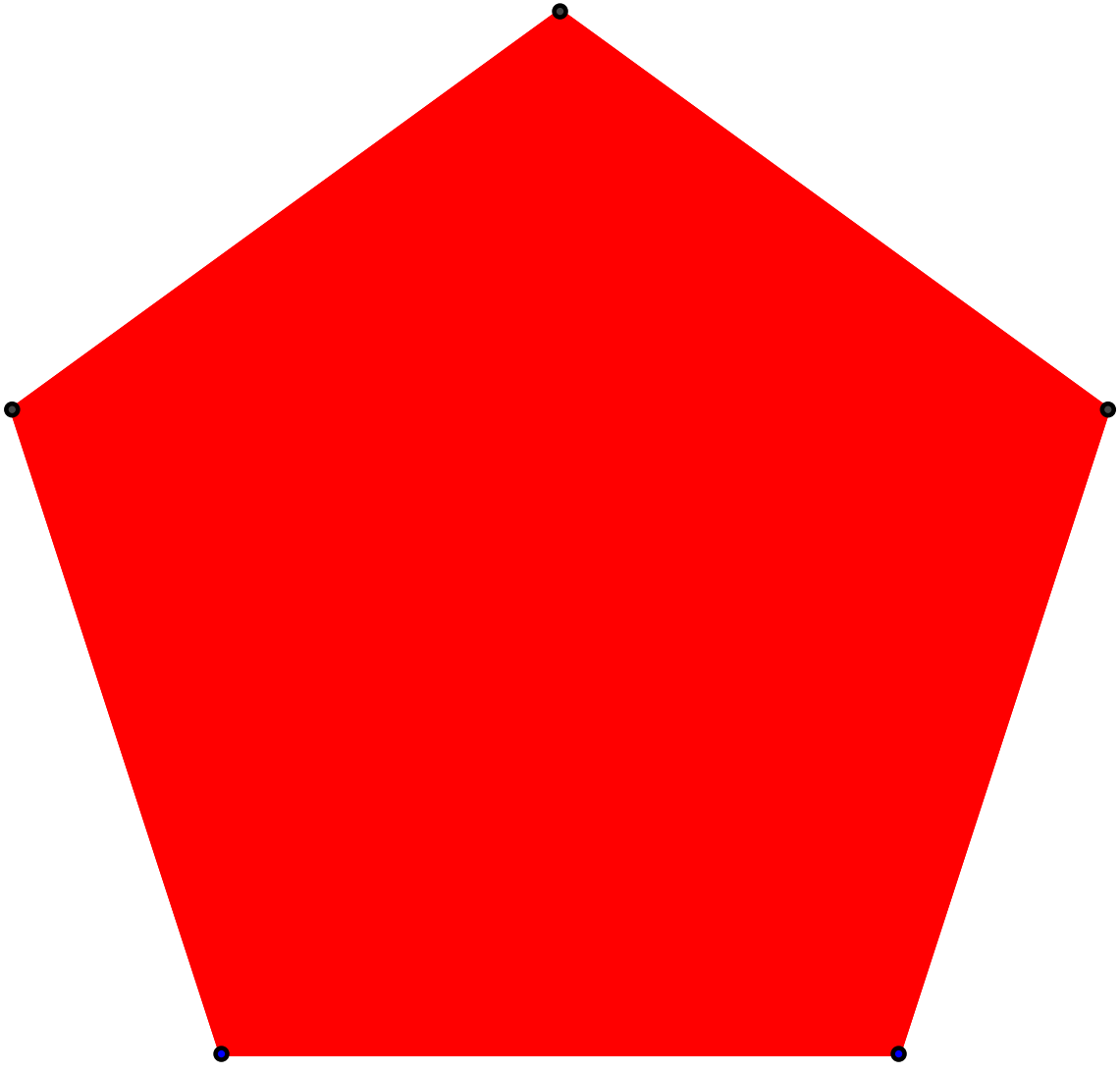
Hexagone avec un centre de symétrie

Des pentagones



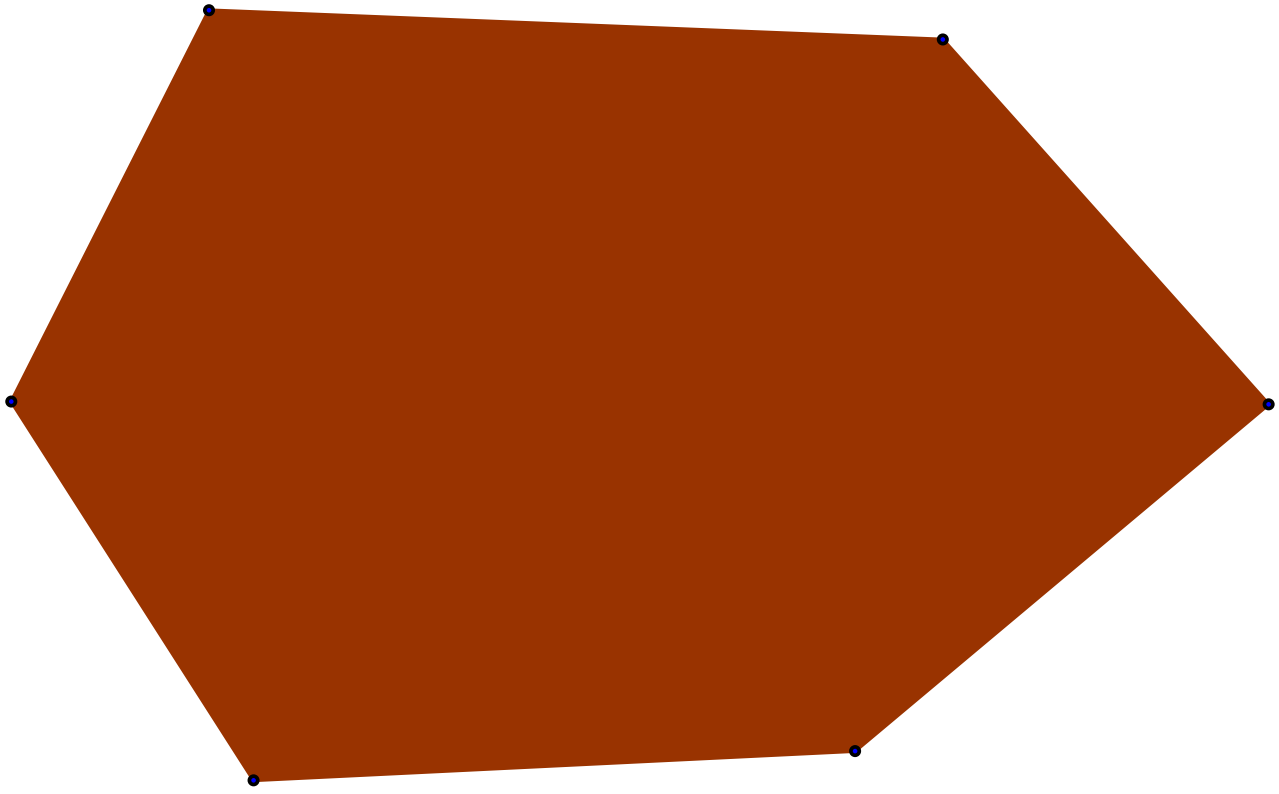


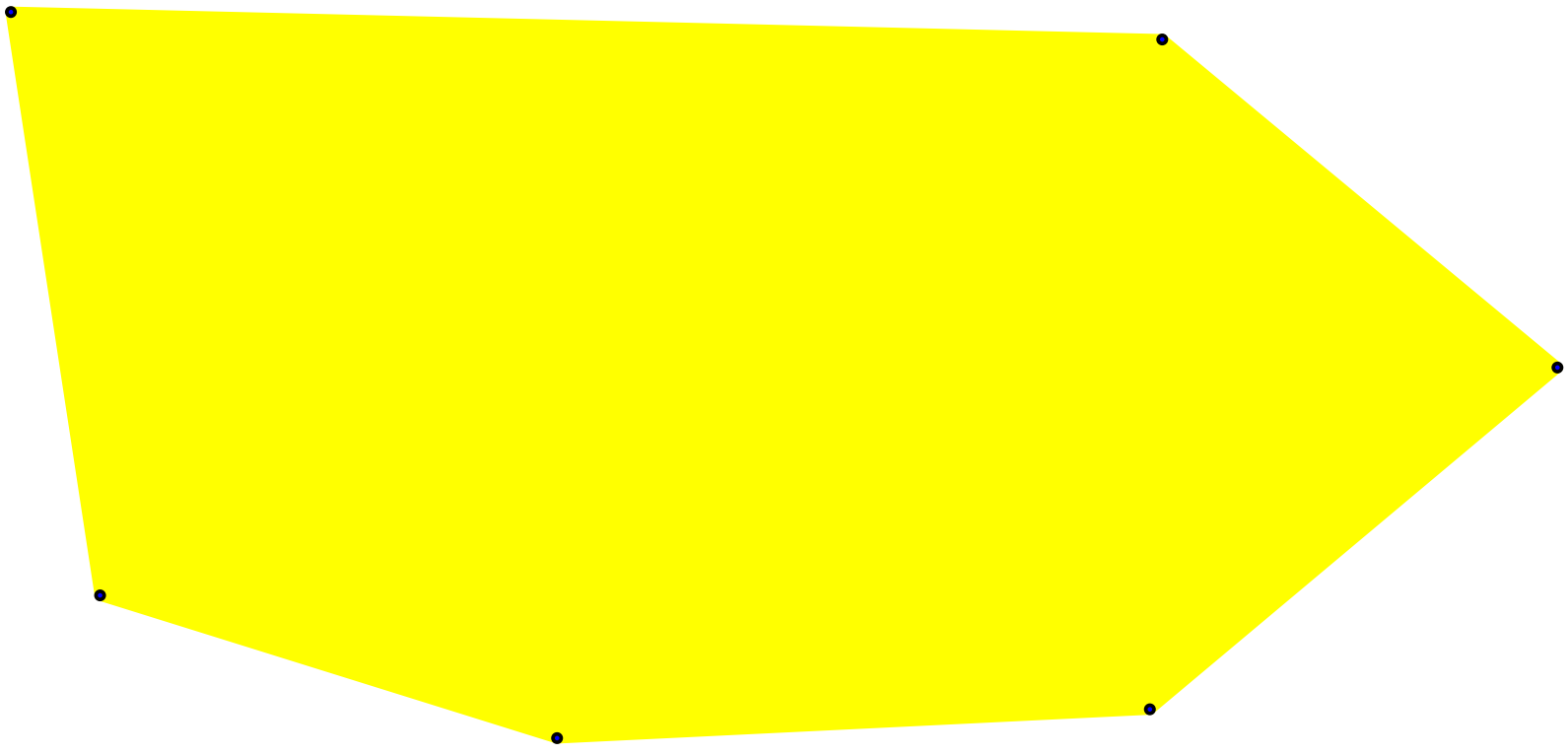


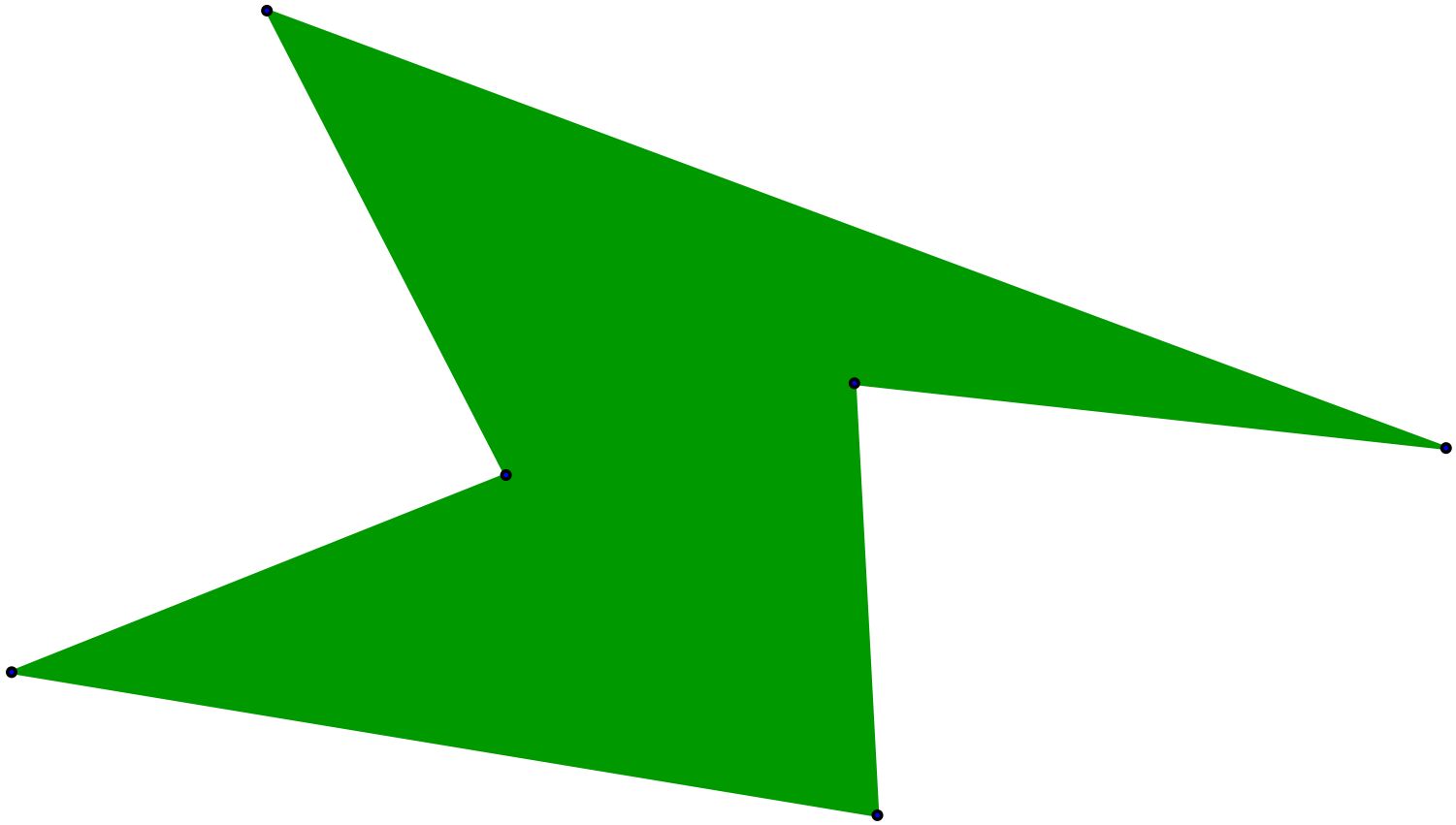


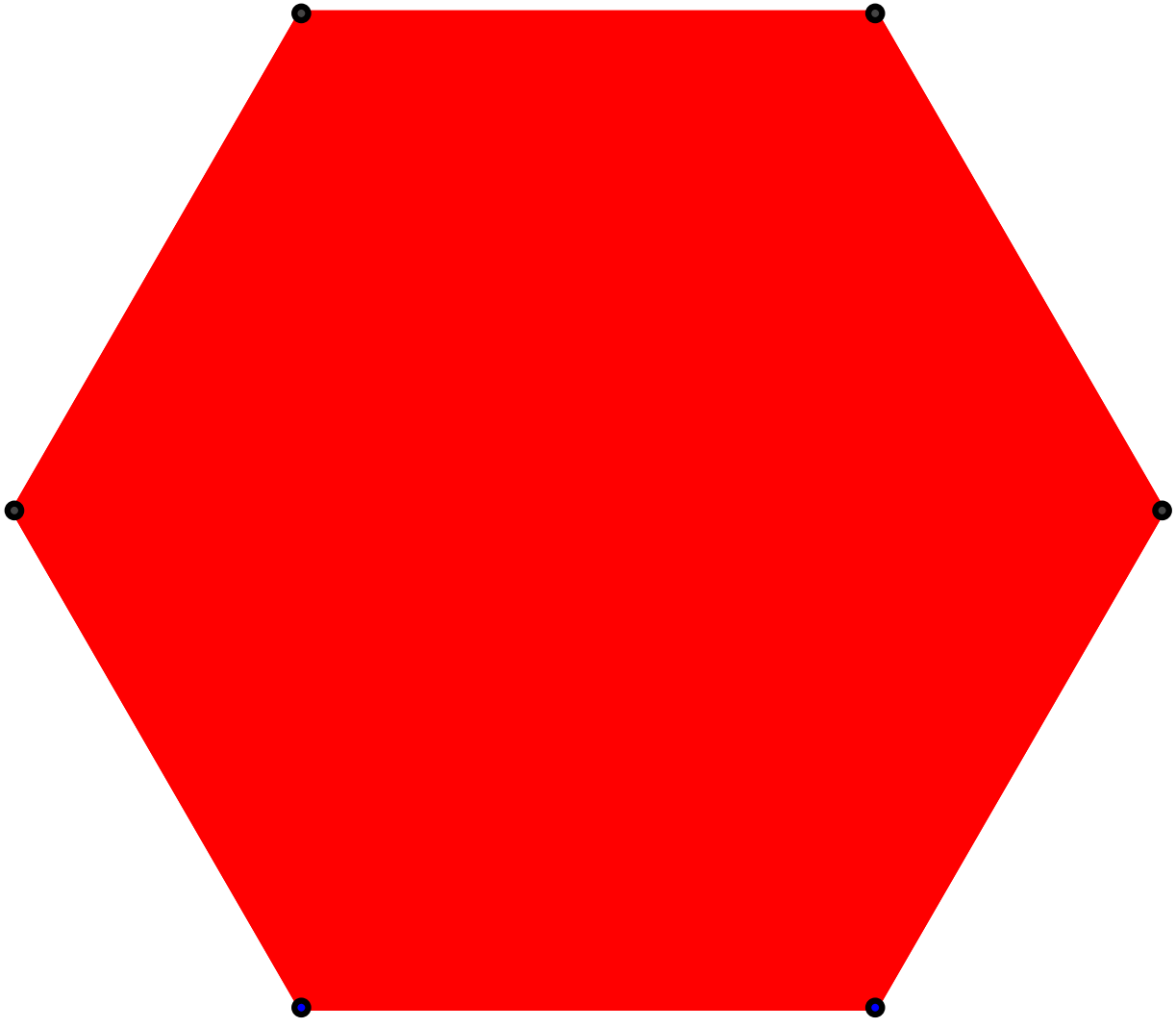
Pentagone régulier

Des hexagones

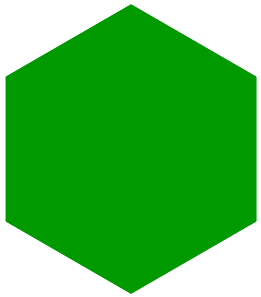


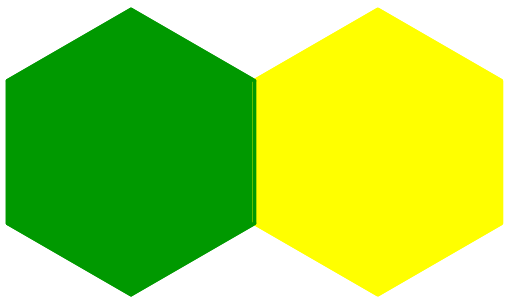


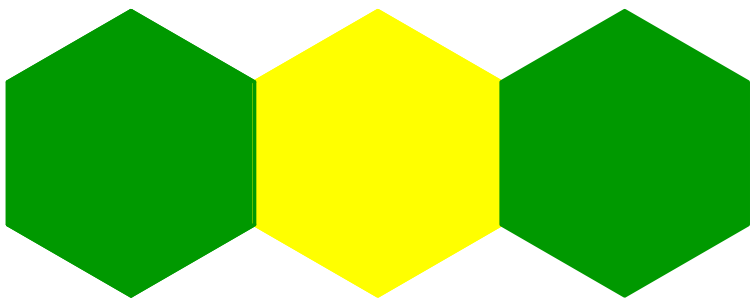


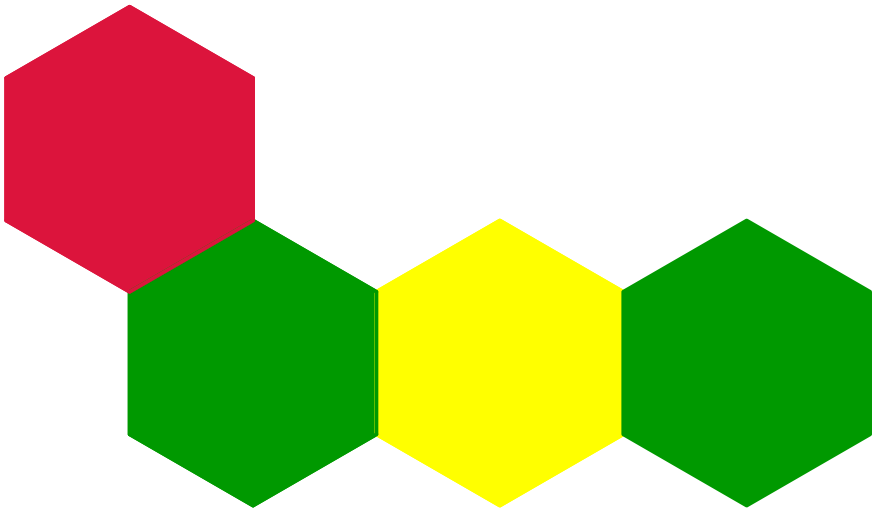


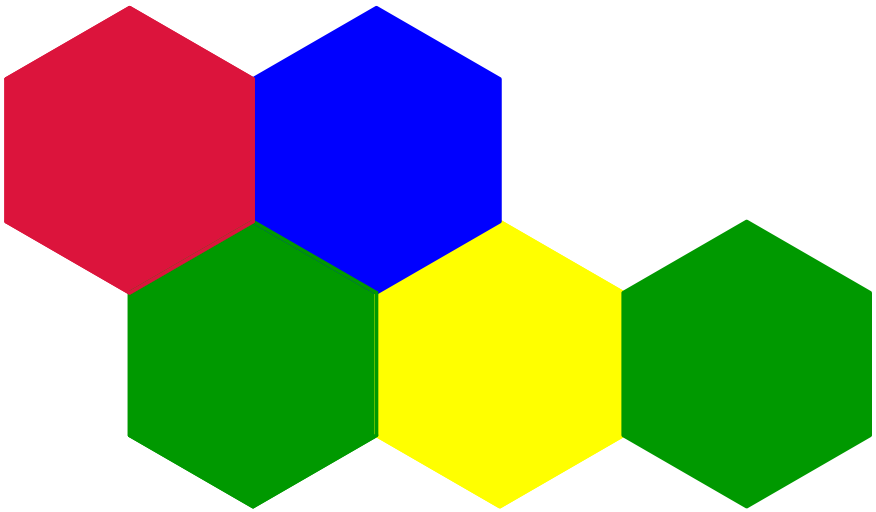
Hexagone régulier

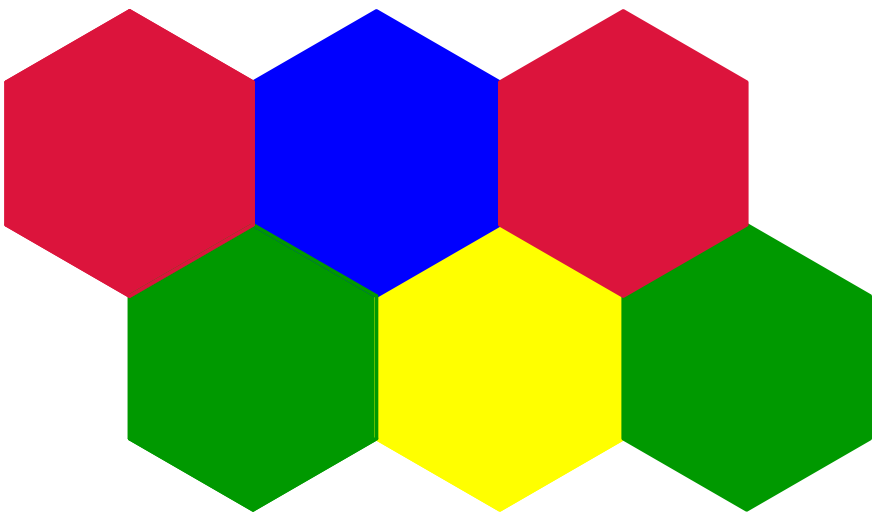


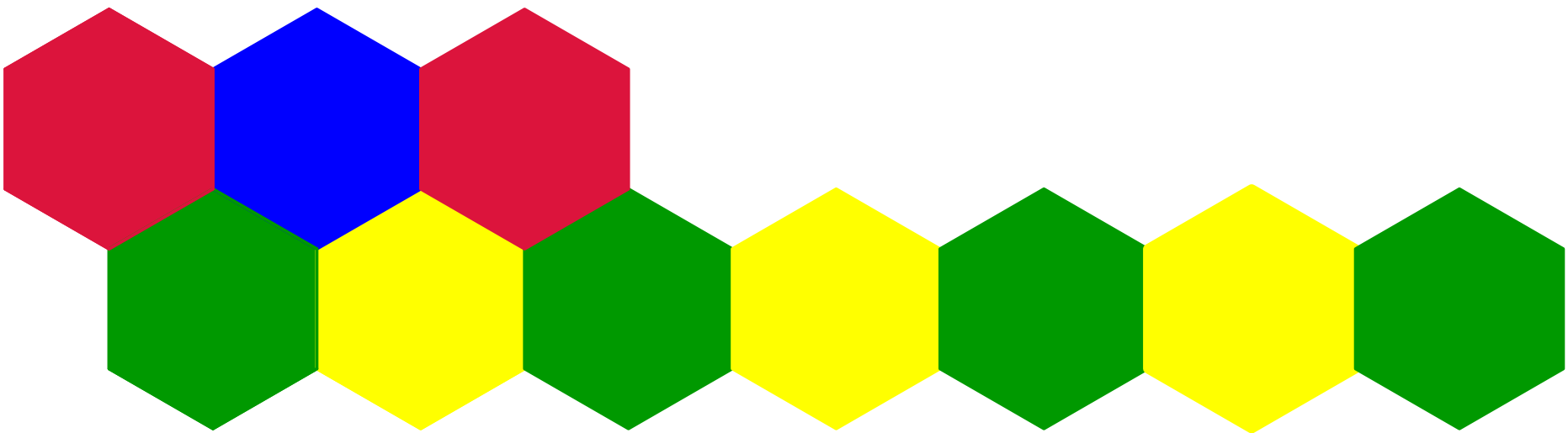


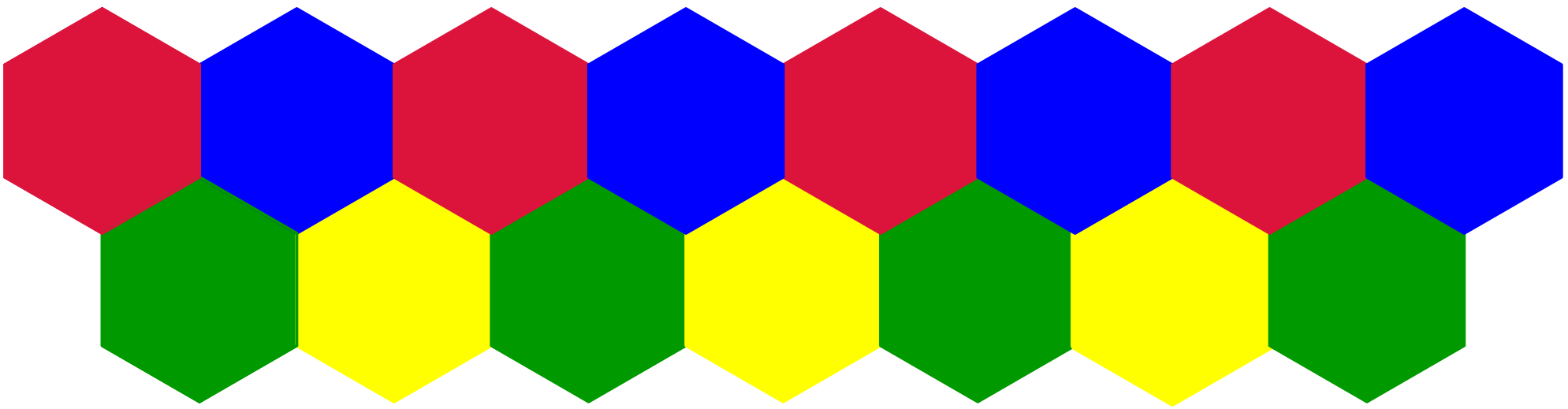


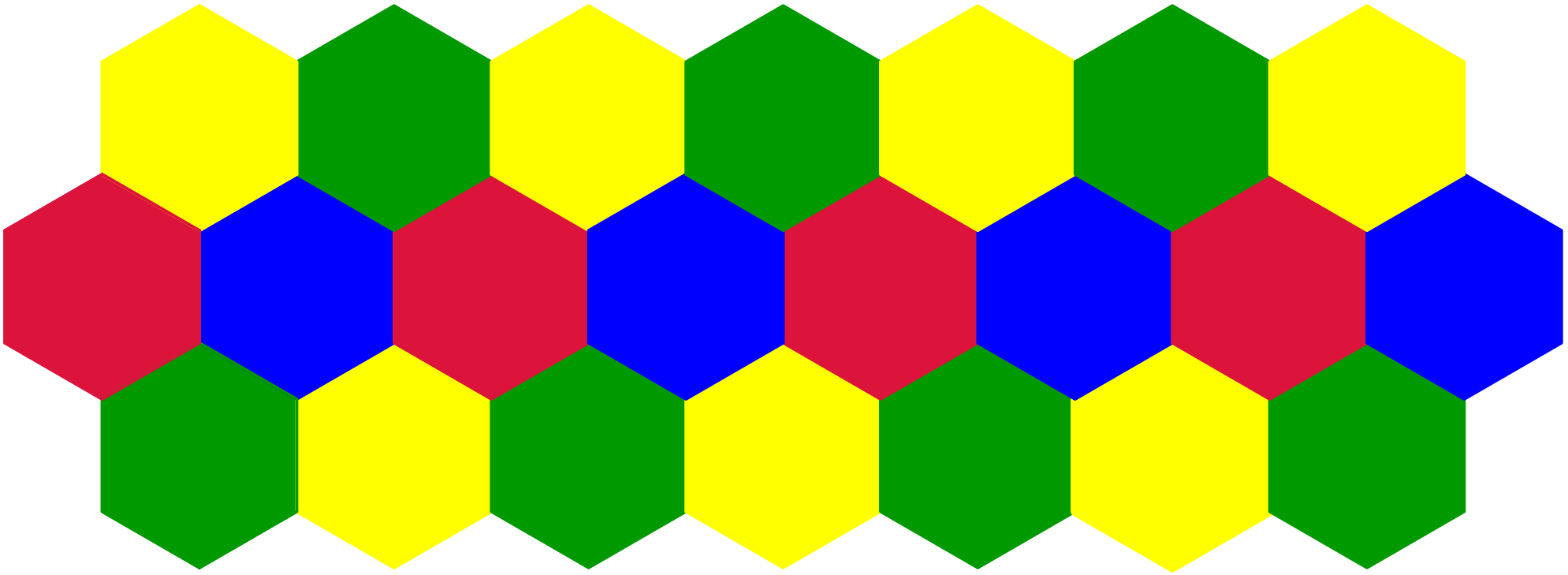




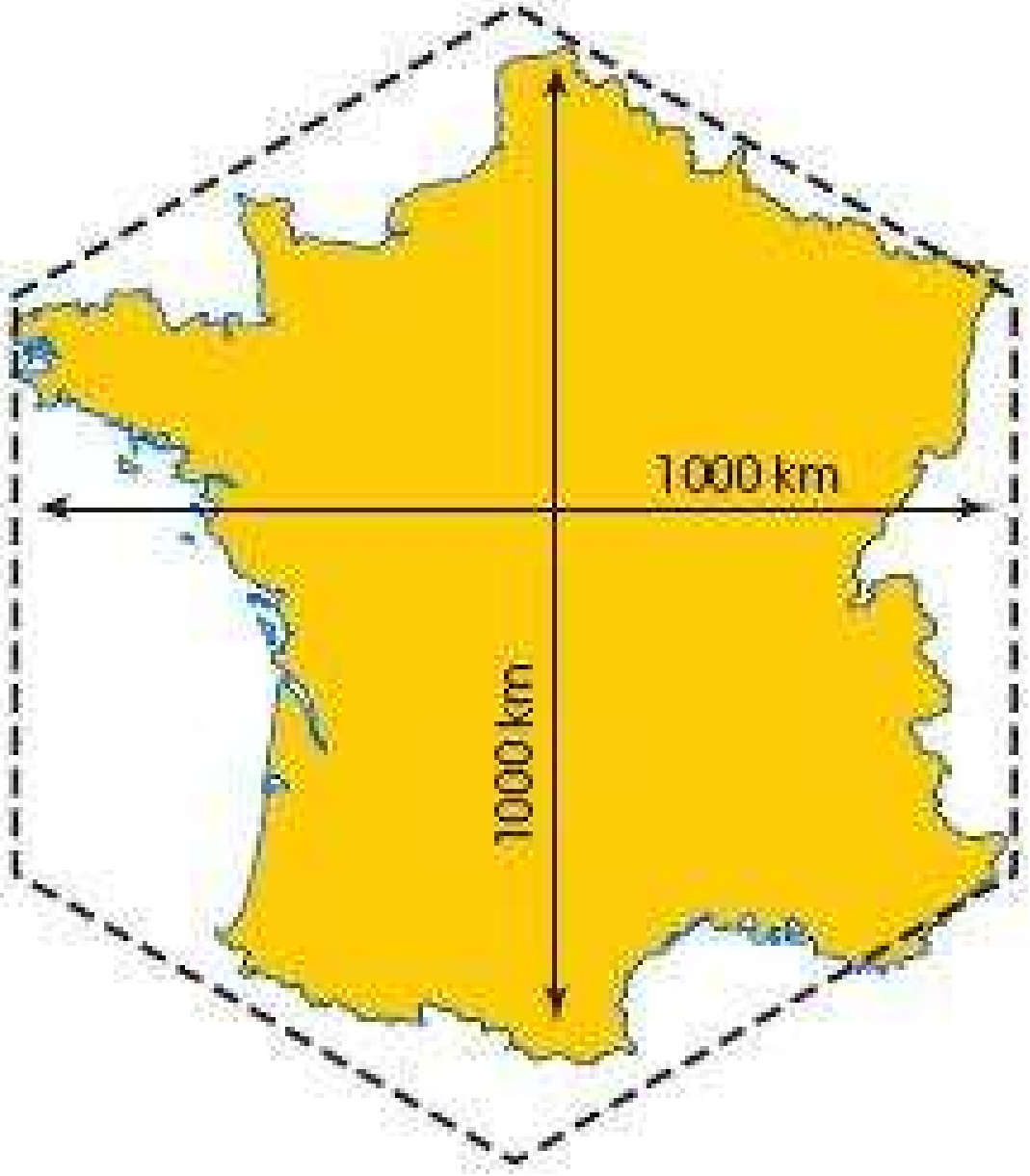








40.000 colonnes hexagonales formant une falaise de 28 mètres de hauteur dans la côte nord de l'Irlande


1000 km
1000 km