

Report of the paper:

On leafwise meromorphic functions with prescribed poles

by Aziz El Kacimi Alaoui

In my opinion this paper is very good and at the high level of quality required by *BBSM*. The paper is very well-written with complete and correct proofs. It contains interesting and useful results in the theory of foliations by Riemann surfaces.

The paper deals with a foliated version of Mittag-Leffler's theorem about the existence of foliated meromorphic functions with prescribed poles defined on a foliation by Riemann surfaces.

The author proves and develops many interesting things but his main result is the following:

Theorem 1 *Let \mathcal{F} be a foliation by Riemann surfaces defined by a differentiable trivial fibration $\pi : M \rightarrow B$. Let $\sigma : B \rightarrow M$ be a continuous section whose image is contained in a \mathcal{F} -relatively compact subset of M . Then for any relatively \mathcal{F} -open set U which contains the image $\sigma(B)$ of σ and any integer s there exists a leafwise meromorphic function $f : U \rightarrow \mathbb{C}$ of class C^s which is non constant along any leaf of (U, \mathcal{F}) and whose set of poles is exactly $\sigma(B)$.*

Remark 1 *Although the author does not state it explicitly it follows by an application of the continuity of Rouché's theorem that if B is connected then the order of the pole $\sigma(b)$ is an integer independent of $b \in B$.*

I must mention here that the author is an expert, and one of the pioneers, of the theory of foliated cohomology in the sense of "transversal cohomology" (which only depends on the transversal holonomy pseudogroup) and "longitudinal cohomology" (along the leaves). The proof of this main result uses the foliated versions of several theorems in analysis and in complex analysis in particular the delta bar foliated operator $\bar{\partial}_{\mathcal{F}}$ and the corresponding $\bar{\partial}_{\mathcal{F}}$ -cohomology for foliations by complex manifolds. The author then develops the foliated Dolbeault-Grothendieck Lemma, cohomology with values in an \mathcal{F} -sheaf, functional (Hilbert) spaces of holomorphic foliated functions (Spaces of \mathcal{F} -holomorphic functions) etc. The proof is very pretty and a *tour de force* of these methods.

The results could have important applications to holomorphic dynamical systems, for instance.

For the above reasons I strongly recommend its publication in *Bulletin of the Brazilian Mathematical Society*.